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Load-Deflection Characteristics and Ultimate Load Capacity of Reinforced Concrete One-way Slab

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Abstract

This study is conducted to investigate and compare the experimental load-deflection characteristics and ultimate load capacity of reinforced concrete one-way slab to the theoretical predictions. Six one-way slabs with dimensions 1000 mm x 150 mm x 50 mm, reinforced with 2-8 mm diameter main bars, were constructed. All specimens tested are simply supported beams subject to center-point load. The Universal Testing Machine is used to apply the load while the deflections of the slabs are measured using a dial indicator.

The study reveals that the experimental deflections are 25% less than the theoretical deflections. The study also shows that the experimental ultimate load capacity of the slabs is 37% greater than the theoretical ultimate load capacity.

Keywords: Load deflection, Reinforced One-way slab

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Introduction

In reinforced concrete construction, slabs are used to provide flat, useful surfaces. A reinforced concrete slab is a broad, flat plate usually horizontal, with top and bottom surfaces parallel or nearly so. It may be supported by reinforced concrete beams (and is usually poured monolithically with such beams), by masonry or reinforced concrete wall, by structural steel members directly by columns, or continuously by the ground. If a slab supported on beams or walls spans a distance more than twice that in the perpendicular direction, much of the load is carried on the short span that the slab may reasonably be assumed to be carrying the entire load in that direction. Such a slab is called a one-way slab.

This study investigates the load-deflection characteristics and ultimate load capacity of simply supported reinforced concrete one-way slab. The deflection and ultimate load capacity of the slab are determined both theoretically and experimentally. The theoretical deflection are determined using the same methods as for beams while the experimental deflection are obtained by constructing a prototype slab and determining the load for each increment of deflection up to failure. The experimental deflection and ultimate load capacity are compared with the theoretical results.

Objective of the Study

The main objective of this study is to compare the experimental results and theoretical predictions of the load-deflection characteristics of a reinforced concrete one-way slab. Also, to compare the experimental ultimate load capacity and the theoretical ultimate load capacity of a reinforced concrete one-way slab.

Review of Related Literature

Flexural Analysis of One-Way Concrete Slabs Reinforced with GFRP Rebars

In a paper entitled, "Flexural Analysis of One-Way Concrete Slabs Reinforced with GFRP Rebars", cracking and deflection of Glass Fiber Reinforced Polymer (GFRP) reinforced concrete structures are analyzed both theoretically and experimentally. Four one-way slabs (2743.2 mm x 457.2 mm x 101.6 mm), consisting of three reinforced with GFRP bars and one reinforced with traditional steel bars were considered. Flexural tests were done on slabs up to failure in static loading condition. The parameters that they considered in the analysis included the type of reinforcement (steel and GFRP), the amount and size of GFRP bars and the thickness of concrete cover. All specimens were tested as simply supported beams subject to a four-point load. A hydraulic jack was used to apply a concentrated load on a steel distribution beam to produce two-point loading condition. Five linear variable differential transformers (LVDT) were used for each specimen to monitor vertical displacements. One LVDT was located at the midspan, two LVDTs were located at quarter-span and two LVDTs were located at the specimen's support to observe settlement, if any. After a series of experiments, the results show good agreement between theoretical predictions and experimental values mainly in the service conditions corresponding to a load level less than 50% of the ultimate load.

Theoretical Considerations

Slabs may be supported on two opposite sides only, in which case the structural action of the slab is essentially one-way the loads being carried by the slab in the direction perpendicular to the supporting beams. For purposes of analysis and design, a unit strip of such a slab cut out at right angles to the supporting beams may be considered as a rectangular beam of unit width, with a depth equal to the thickness of the slab and a span equal to the distance between supported edges. This strip can then be analyzed by the methods that were used for rectangular beams-the bending moment being computed for the strip of unit width. The load per area on the slab becomes the load per unit length on the slab strip. Since all the load on the slab must be transmitted to the two supporting beams, it follows that all the reinforcement should be placed at right angles to these beams, with the exception of any bars that may be placed in the other direction to control shrinkage and temperature cracking. A one-way slab consists of a set of rectangular beams side by side.

In the design of reinforced concrete one-way slab, slab deflections may be calculated by the same method as for beams. It can be expressed in the general form $\Delta = f$ (loads, spans, supports) / EI where EI is the flexural rigidity and f (loads, spans, supports) is a function of the particular load, span, and support arrangement.

Modulus of Rupture

The modulus of rupture, f_r is a measure of the tensile strength of concrete under pure flexural stresses. Most design codes have adopted the modulus of rupture as a representative of concrete cracking strength in deflection computations. Design codes provide different prediction models for the modulus of rupture. Most of these models relate the modulus of rupture to the square root of the concrete compressive strength, f'_c in order to reflect the disproportional increase of the modulus of rupture with respect to the increase in the compressive strength. The ACI Code contains the recommendation that the modulus of rupture f_r be taken to equal $7.5\sqrt{f'c}$ for normal weight concrete, and that this value be multiplied by 0.85 for "sand light-weight" and 0.75 for "all-lightweight" concretes, giving values of $6.4\sqrt{f'c}$ and $5.6\sqrt{f'c}$ respectively for those materials.

Cracking and Crack Development

As long as the tensile stress is smaller than the modulus of rupture, no tension cracks an develop. The strain and stress distribution is essentially the same as in an elastic, homogeneous beam. The only difference is the presence of another material, the steel reinforcement. In elastic range, for any given value of strain, the stress in the steel is n times that of the concrete. However, when the tension stress, f_{ct} exceeds the modulus of rupture, cracks form. If the concrete compression stress is less than approximately $f'_c/2$ ant the steel stress has not reached the yield point, both the material continue to behave elastically, or very nearly so.

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At this stage, for simplicity and with little if any error, it is assumed that that tension cracks have progressed all the way to the neutral axis and that sections plain before bending are plane in the bent member.

Deflection Behavior of Slab (Nawy, 1996)

The load-deflection relationship of a reinforced concrete slab is basically trilinear, as idealized in Figure 2.3.

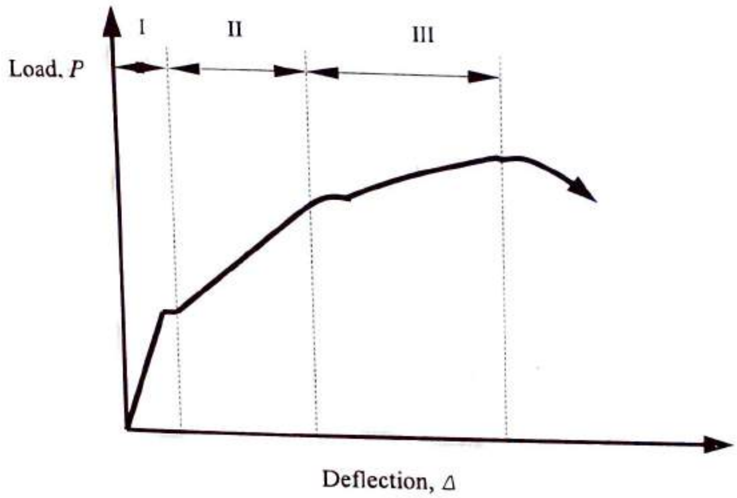


Figure 2.3 Slab Load-deflection Relationship

Region I is the precracking stage, where the structural member is crack-free. It is essentially a straight line defining full elastic behavior. The maximum tensile stress in the slab in this region is less than its tensile strength in flexure, that is, less than the modulus of rupture, f_r of concrete. Region II is the postcracking stage, where the structural member develops acceptable controlled cracking both in distribution and width. Region III is the postserviceability cracking stage, where the stress in the tension reinforcement reaches the limit state of yielding.

Properties of Stress-Strain Curve (Everard and Tanner, 1996)

Figure 2.4 presents the usually accepted plot of the stress-strain diagram for concrete subjected to axial load and flexure which was developed statistically. Certain properties of the curve must be described;

(a.) The tangent to the curve at its origin is called the *initial tangent modulus of elasticity*, E_{ci}, psi. (b.) A line drawn from the origin to a point on the curve at which $f_c = 0.45f'_c$ is called the *secant modulus of elasticity*, E_{cs}, psi. In general practice, it is usually used as the modulus of elasticity of concrete and is simply referred to as E_c. (c.) For lightweight aggregate concrete, the initial slope is somewhat less than that for normal weight concrete. The maximum stress occurs for larger strain values for lightweight concrete when f'_c is the same for both types of concrete.

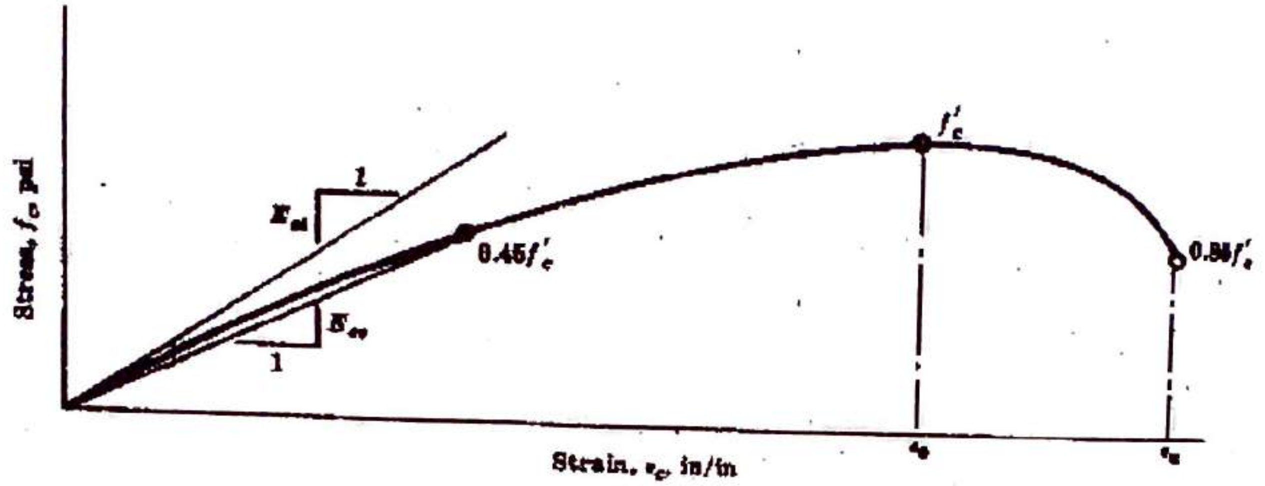


Figure 2.4 Stress_Strain Diagram

Methodology

Equipment Preparation and Materials Processing

A. For the production of concrete specimens in this study, three sacks of coarse aggregates and two sacks of fine aggregates are taken from a hollow block factory located in Tibanga, Iligan City. They originally came from a quarry site in Mandulog, Hinaplanon, Iligan City. They are manually washed using a basin to minimize the deleterious materials (e.g. silt, pieces of wood) which affect the strength of concrete.

B. The coarse aggregates are sieved using 3/4" screen to have a nominal size of 20 mm diameter. They are further sieved to be retained in # 4 screen.

C. The fine aggregates are sieved using # 4 screen.

D. A sack of Type 1 Portland cement is used in the study. The water for mixing is taken from MSU-IIT distribution lines supplied by Iligan City Water Works District.

E. Six slab molds with an inner dimension of 1000 mm x 150 mm x 50 mm are prepared to make the reinforced concrete one-way slab specimens. 1/8" plywood is used as the base of the mold while 1/2" thick lumber is used for the sides of the mold.

F. The reinforcing steel bars are cut using a steel saw. The main bars and temperature bars are 92.2 cm and 9 cm length, respectively. They are joined together by using tie wires.

G. The dial indicator, which is mounted on a tripod, is used to measure the deflection of the specimens.

Pertinent ASTM Standards Used In Relation To Aggregates

1. ASTM C 136 – Test Method for Sieve Analysis of Fine and Coarse Aggregates
2. ASTM C 29 – Unit Weight Determination Test
3. ASTM C 127 – Test Method for Specific Gravity and Absorption of Fine and Coarse Aggregates

Mixing and Casting of Slab and Concrete Cylinders

Mixing of concrete is done with cement-aggregate proportion of 1:2:3 by weight. The preparation of aggregates is initiated by weighing them according to the prescribed cement-aggregate ratio. A 0.5 water-cement ratio is used. Since the aggregates are in their saturated surface-dry condition during the mixing, no adjustment of water is done.

A total of 4 batches of mixtures are made to prepare the 6 reinforced concrete one-way slab specimens and the 4 concrete cylinder specimens. Slump test is conducted for each batch of mixture.

Casting the Reinforced Concrete One-Way Slab Specimens

Six reinforced concrete slabs with dimensions 1000 mm x 150 mm x 50 mm are prepared for this study. The slabs have a clear span of 800 mm. However, 100 mm length on each side is extended beyond the support. The slabs are reinforced with 2- 8 mm diameter main bars at 75 mm o.c. and 3- 6 mm diameter temperature bars at 400 mm o.c.

After mixing the concrete and the slump, a test is conducted, the concrete is placed in the slab molds. The molds are used in its saturated surface dry condition by soaking it in water for 24 hours and wiping it with cloth before using. The first step is to make a mark 2 cm above the bottommost of the mold. The fresh concrete is then placed into the mold up to this level. This serves as the concrete cover up to the reinforcing steel bars from the bottom. Next, the prepared reinforcing steel bars are laid with its proper orientation. Finally, the slab mold is filled with the concrete mixture up to the top and levelled using a straight edge. The sides of the mold are then tapped to minimize the voids and to ensure close contact with forms and reinforcement.

Casting the Concrete Cylinder Specimens

Four concrete cylinder specimens are prepared using the ASTM standard size of 6" by 12" using four concrete cylinder molds. Before using the molds, oils were applied on them to prevent the concrete from sticking into the molds. The casting is done in accordance with ASTM C31 which is the "Standard Method for Making Concrete Test Specimens in the Field".

Curing

All specimens are removed from their molds after 20-24 hours. The researcher is only concerned on the 28-day compressive strength of concrete, so the curing period lasts for 28 days. After taking the specimens out of their molds, they are soaked in water in the curing tub. At the end of the 28th day, the samples are all taken out of the water and are allowed to dry for testing.

Testing of Specimens

Determining the Rate of Straining

All specimens are loaded at the same rate of straining. The rate of straining is determined by getting the deflection per second (mm/s) of the Universal Testing Machine (UTM). This is done by putting the dial indicator on the table of the UTM. As the UTM is loaded, the time in which the dial indicator completes one revolution is determined using a stopwatch. The rate of straining is chosen in such a way that the load and the deflection readings can be made.

Compression Test of Concrete Cylinder Specimens

Compression test is conducted on each concrete cylinder specimen using the UTM. The specimen is loaded at the chosen rate of straining. A person is assigned to record the load while the researcher watches carefully the reading at the dial indicator. The recordings of the loads are made for every 0.1 mm increment of deflections.

The load and deflection data are used to obtain the complete stress-strain diagram of concrete in this study. The load is transformed to stress by dividing it by the cross-sectional area of the concrete cylinder specimen (P/A). The deflection is transformed into strain by dividing it by the original height of the concrete cylinder ($\epsilon = \delta/L$). The compressive strength, f'_c which is the average maximum stress is used for the computation of the theoretical deflection and the ultimate load capacity of the reinforced concrete one-way slab specimens. The transformed stress-strain values and the stress-strain diagram are shown in Table 4.6 and Figure 4.4, respectively.

Density of Concrete

The density of concrete is determined by getting the weight of the concrete cylinder specimen and divided by its volume. In this study, the obtained average density of the specimens are used to determine the self-weight of the reinforced concrete one-way slab and for the theoretical computations of the modulus of elasticity of concrete.

Testing of the Reinforced Concrete One-Way Slab Specimens

The reinforced concrete one-way slab specimens are tested with center-point type of loading. The deflection is measured by doing the same procedure as the concrete cylinder specimens. But this time, the deflection is read at 0.5 mm increment. The experimental deflection and ultimate load capacity of the specimens are compared with the theoretical results.

Yield Strength, Tensile Strength and Elongation of Reinforcing Steel Bars

The yield and tensile strength of the reinforcing steel bars used in the study are determined using the UTM. Three specimens, 2 ± 500 mm long, are tested. The yield strength is determined by first determining the yield point. It is determined by the Halt of the Pointer Method. This is done in accordance with ASTM A370-92, which is the Standard Test

Methods and Definitions for Mechanical Testing of Steel Products. The yield point is then transformed into yield strength by dividing it by the original cross-sectional area of the reinforcing steel bar specimen. This average stress is used as the f_y for the theoretical computations. The tensile strength is determined by dividing the maximum load of the specimen by its original cross-sectional area.

The total strain of each reinforcing steel bar specimen is determined. Before testing, the original gage length of the steel bars is measured to the nearest 0.05 mm using a steel tape. After testing, the ends of the fractured specimens are carefully fitted together. The elongated length is again measured using the steel tape. The elongation divided by the original gage length is the strain of the specimen.

Theoretical Deflection of Reinforced Concrete One-Way Slab

The theoretical deflections are computed using the following procedure and formulas: a. Determination of design data: A_s, f'_c and E_s.

A_s, area of steel is determined using the nominal diameter

f'_c is based from compression test of concrete cylinders

E_s, standard modulus of elasticity of steel taken as 200,000 Mpa

b. Calculation modulus of concrete E_c,

$$E_c = 0.043 w_c^{1.5} \sqrt{f'c}$$

where w_c = density of concrete in kg/m^3

f'_c = compressive strength of concrete

c. Calculation of modular ratio $n = E_s / E_c$;

where $E_s = 200{,}000$ MPa

d. Calculation of gross moment of inertia, I_g and cracking moment, M_{cr}

$$I_g = bh^3/12$$

where b = width of the slab

h = thickness of slab

$$M_{cr} = I_g f_r / y_t$$

where f_r = modulus of rupture of concrete

$$f_r = 0.7\sqrt{f'c}$$

y_t = the distance of the extreme tension fiber from the uncracked section center of gravity

$$y_t = h/2$$

e. Calculation of axis depth, c of the cracked section and the cracking moment of inertia, I_{cr}

The value of c can be obtained by solving the quadratic equation,

$$bc^2/2 + nA_sc - nA_sd = 0$$

$$I_{cr} = bc^3/3 + nA_s(d-c)^2$$

f. Calculation of effective moment of inertia, I_e

$$I_e = (M_{cr} / M_a)^3 I_g + [1 - (M_{cr} / M_a)^3] I_{cr} \quad \leq \quad I_g$$

where M_a - the maximum moment for particular load (Refer to Figure 3.16)

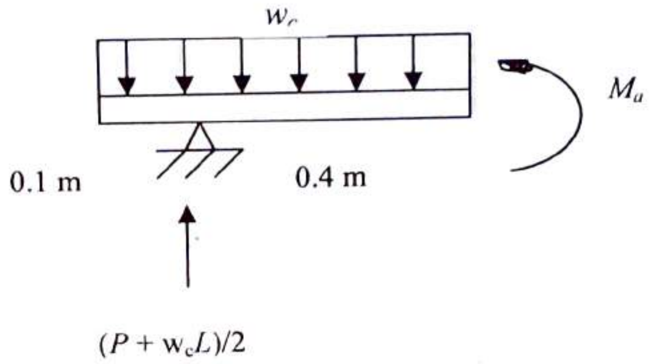


Figure 3.16. Free-body Diagram of Slab

g. Calculation of the deflection, Δ

for simply supported,

$$\Delta = Pl_n^3/48\ E_cI_e + 5w_cL^4/\ 384\ E_cI_e$$

if $M_a < M_{cr}$, the slab will not crack under the applied load

use $I_e = I_g$

where P- the applied load at midspan

l_n - clear span length

L = total length of slab

Theoretical Ultimate Load Capacity of Reinforced Concrete One-Way

Slab

The theoretical ultimate load capacity of the reinforced concrete one-way slab is computed using the following procedure and formulas:

a. Calculation of the ultimate moment, M_n at which the slab will fail by yielding of tension steel

$$M_n = \rho f_y b d^2 (1 - 0.59 \rho f_y / f'c)$$

The computed theoretical value of M_n is 0.53 kN-m.

b. Calculation of the ultimate load capacity, P_n (Refer to Figure 3.16)

The ultimate moment, M_n obtained from (a.) is substituted in the formula,

$$M_n = (P_n + w_c L)/2\,(0.4) - w_c(0.5^2)/2$$

$$M_n = 0.2(P_n + w_c L) - 0.125 w_c$$

$$(M_n + 0.125 w_c)/0.2 = P_n + w_c L$$

Finally,

$$P_n = 5M_n + 0.625 w_c - w_c L$$

The computed theoretical value of P_n is 2.6 kN.

The average of the experimental values is compared with the theoretical values. Comparisons between the experimental and theoretical results are shown in Table 4.12.

Results and Discussions

Sieve Analysis Result

Sieve Analysis of Fine Aggregates

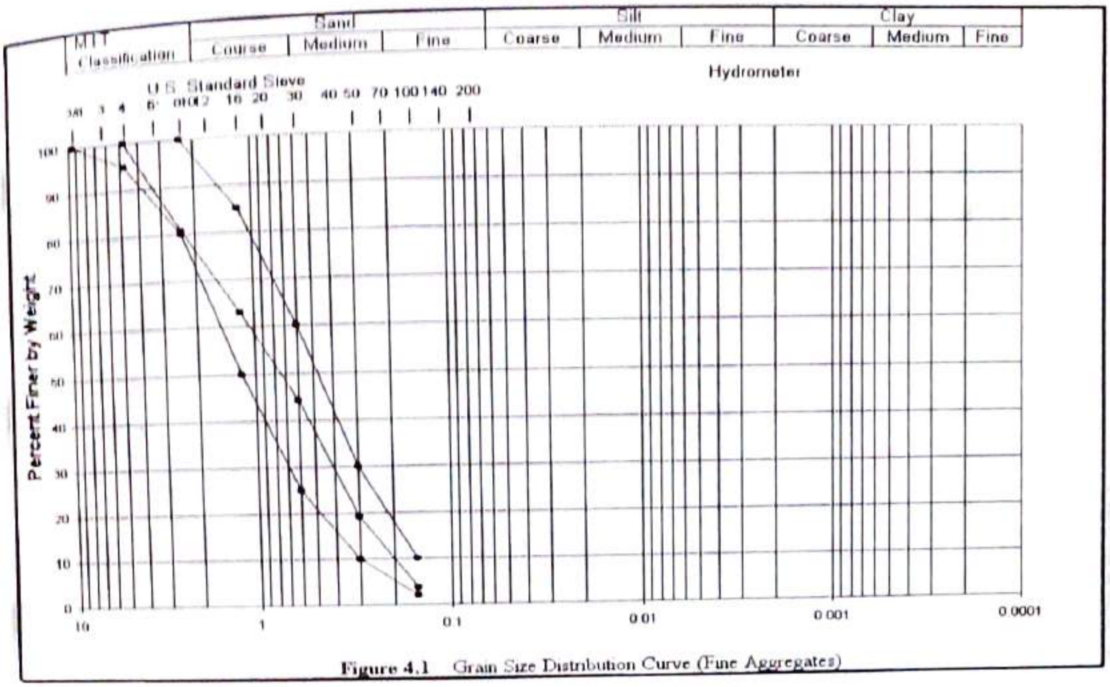


Figure 4.1 Grain Size Distribution Curve (Fine Aggregates)

The result of sieve analysis of fine aggregates (sand) conforms with ASTM C 33-90.

Sieve Analysis of Coarse Aggregate

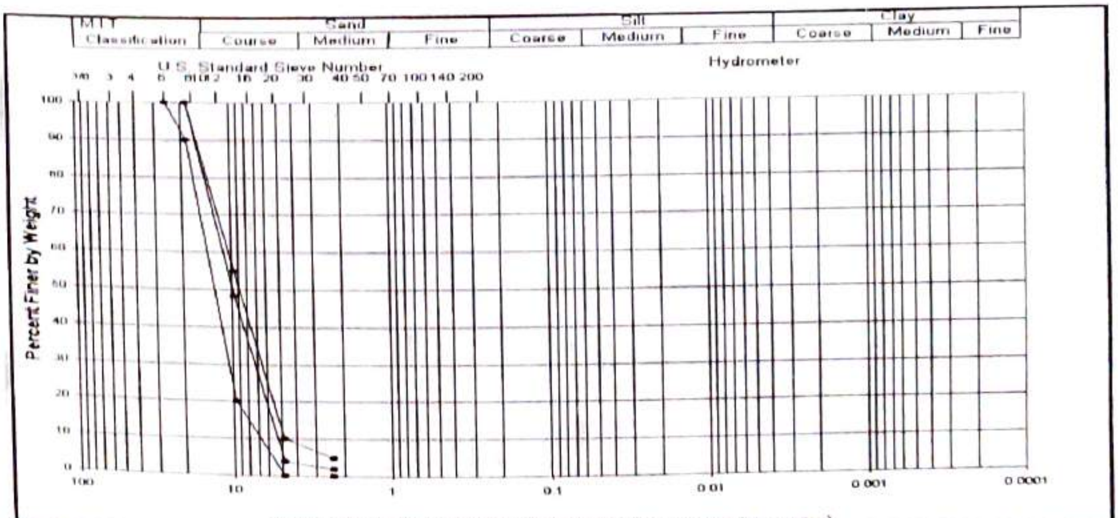


Figure 4.2 GRAIN SIZE DISTRIBUTION (Coarse Aggregates)

The result of sieve analysis of coarse aggregates (gravel) conforms with ASTM C 33.

Compression Test

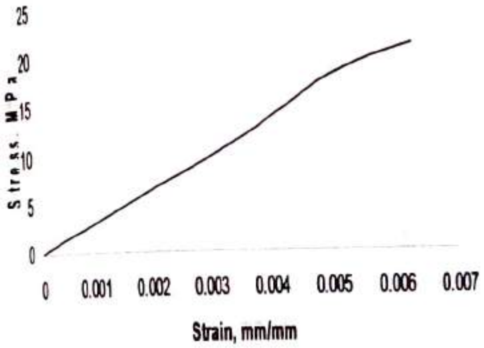
The time in which the dial indicator completes one revolution is 23 seconds. So, the rate of straining that is used is 1 mm/ 23 sec or 0.04 mm/sec. This is applied for all the concrete and slab specimens.

The load for every 0.1 increment of deflection is recorded.

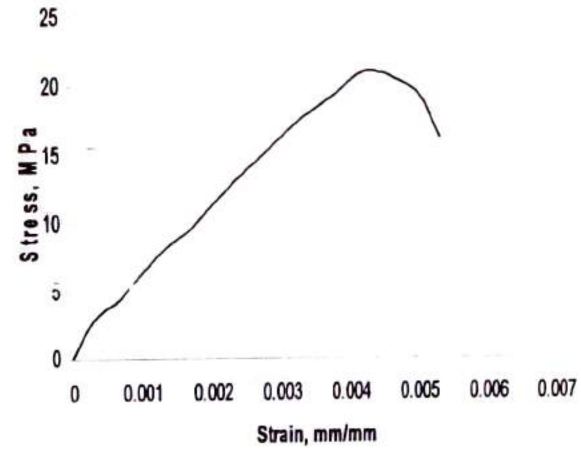
The load and deflection data of the concrete cylinder specimens are transformed to stress and strain, respectively.

Table 4.6. Stress and Strain of the Concrete Cylinder Specimens

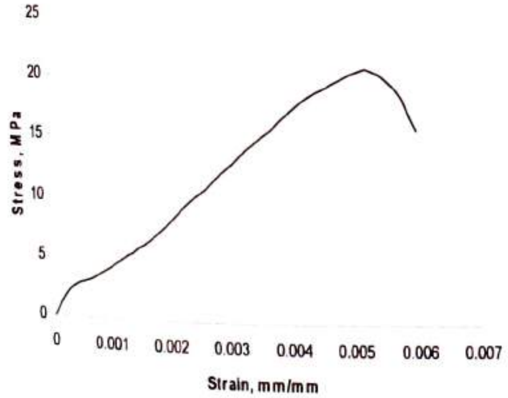
Strain (mm/mm)	Stress (MPa)			
	Specimen 1	Specimen 2	Specimen 3	Specimen 4
0.0003		2.89	2.41	2.97
0.0067		4.47	3.28	3.90
0.0010		6.40	4.10	4.85
0.0013		8.04	5.29	6.28
0.0017	5.15	9.79	6.62	7.64
0.0020		11.49	7.95	9.42
0.0023		13.02	9.56	10.81
0.0027		15.00	11.09	12.31
0.0030		16.47	12.62	13.89
0.0033	10.30	17.94	14.12	15.39
0.0037		19.35	15.68	16.92
0.0040		20.66	17.20	18.00
0.0043		21.22	18.48	19.21
0.0047		19.66	19.52	20.54
0.0050	17.00	16.41	20.32	19.01
0.0053			20.51	16.10
0.0057			18.96	
0.0060			15.68	
0.0063	19.95			



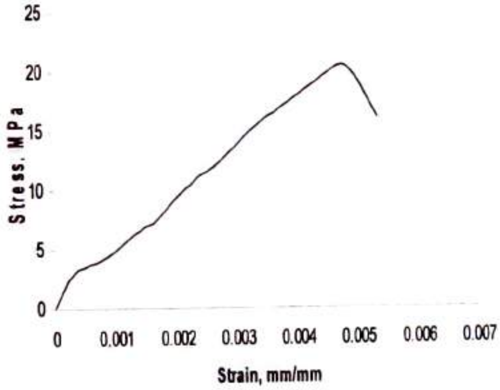
(a) Cylinder 1



(b) Cylinder 2



(c) Cylinder 3



(d) Cylinder 4

Figure 4.4 Stress-Strain Diagrams of the Concrete Cylinders

From the figure above, the stress-strain diagrams of concrete samples indicate three distinct ranges. First is the initial range which is up to 0.0003 mm/mm strain. In this range, the stress and strain are very nearly linear. Second, is the intermediate range which is up to an average strain of 0.005. In this range, there is an increasing curvature, ultimately reaching a point of maximum stress. And lastly, the final range in which strain continues to increase while the load-carrying capacity decreases. The experimental modulus of elasticity of the samples which is the secant modulus of elasticity, is obtained from the stress-strain curves.

Modulus of Elasticity and Compressive Strength of Concrete

Cylinder No.	Initial Modulus of Elasticity (MPa)	Modulus of Elasticity (MPa)	Compressive Strength (MPa)
1		3,452.88	19.95
2	9,633.33	5,787.27	21.22
3	8,033.33	4,120.31	20.51
4	9,900.00	4,691.88	20.54
Average	9,188.89	4,513.09	20.56

Density of Concrete

The density of concrete, w_c, is the weight of the concrete cylinder specimen divided by its volume.

Density of Concrete

Specimen	Weight (kg)	Volume (m^3)	Density (kg/m^3)
1	13.000	0.0053	2,452.17
2	12.970	0.0053	2,446.51
3	13.045	0.0053	2,460.65
4	13.000	0.0053	2,452.17
Average	13.004	0.0053	2,452.88

From the result above, the unit weight of concrete is determined by multiplying the average density by 9.81 m/s^2. The unit weight is then multiplied by the dimension of the slab to obtain its self-weight as shown below.

Self-weight: $w_c = 2,452.88\ (9.81) = 24.06\ kN/m^3\ (0.15\ m)(0.05\ m)$

$w_c = 0.18\ kN/m$

The average density is also used for computing the theoretical modulus of elasticity of the concrete.

$E_c = 0.043\ w_c^{1.5}\ \sqrt{f'c}$; w_c - density of concrete

$w_c = 2452.88\ kg/m^3$

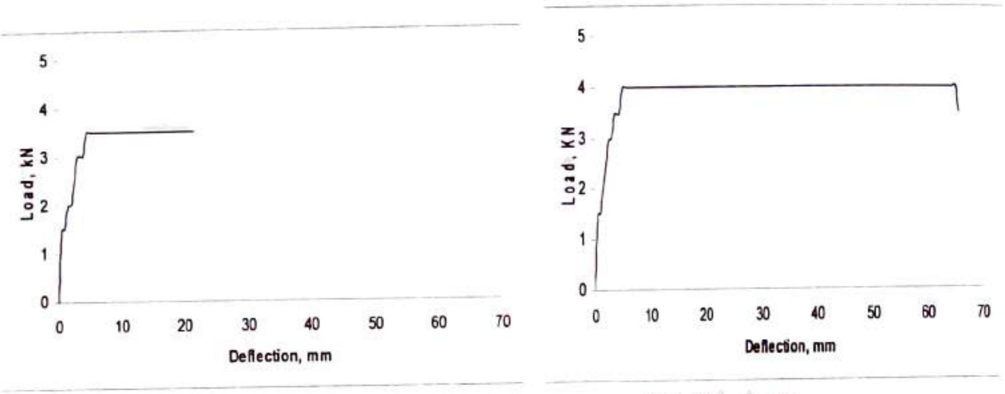
$E_c = 0.043(2452.88)^{1.5}\sqrt{20.56}\ =\ 23,686.15\ MPa$

Load-Deflection Relationship of Reinforced Concrete One-Way Slab

Experimental Deflections

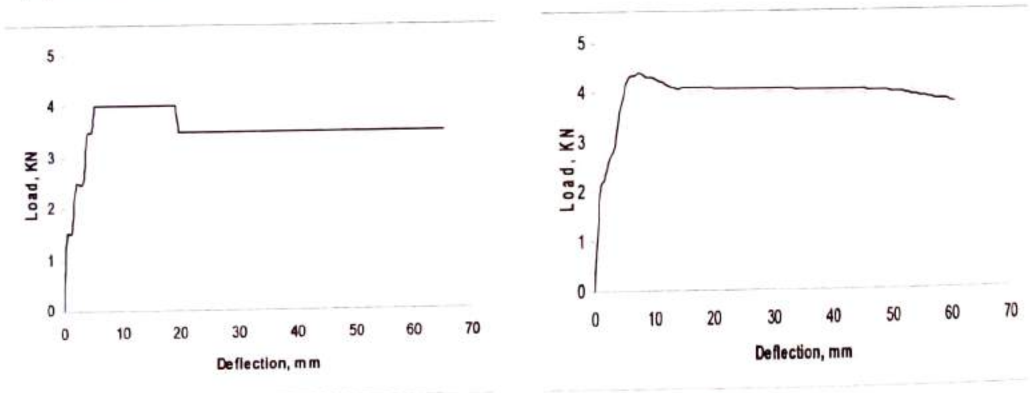
Figure plots the load versus deflection results of the reinforced concrete one-way slabs in this study.

In this test, the load readings from UTM for slabs 1, 2 and 3 are rounded-off values of 1 decimal place only. This is because the UTM was not adjusted. During the test, the UTM was set-up in which the maximum load that can be applied on the specimen is 1000 kN. This load is much greater than the actual load capacity of the slab. Thus, the obtained load-deflection relationship is rough. On the other hand, the load readings for slabs 4, 5 and 6 are more accurate since the set-up of UTM was corrected and adjusted to a maximum load of 20 kN. The UTM registered loads up to 2 decimal places.



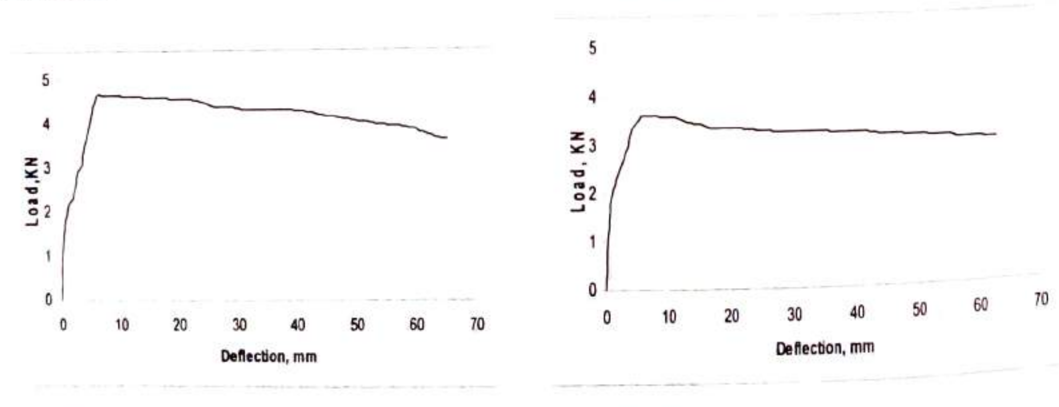
(a) Slab 1

(b) Slab 2



(c) Slab

3 (d) Slab 4



(e) Slab 5

(f) Slab 6

Figure 4.6. Load-Deflection Relationship of Reinforced Concrete One-Way Slab

The load-deflection relationships of all slabs are very nearly similar. The curves agree with the theoretical load-deflection relationship. The ultimate loads are 4.0 kN, 4.0 kN, 4.39 kN, 4.66 kN, and 3.62 kN with an average of 4.13 kN.

Theoretical Deflections

Table 4.10 shows the theoretical midspan deflection of slab up to the ultimate load. Values found in Table 4.10 are computed using the Microsoft Excel.

Table 4.10.Theoretical Deflection of Reinforced Concrete One-Way Slab

SLAB 1			
Load (kN)	*Ma* (N-mm)	*Ie* (mm^{4})	Theoretical Deflection (mm)
0.0	0.0	0.0	0.0
1.5	313500	582980.24	1.23
2.0	413500	396390.21	2.37
2.5	513500	327521.60	3.56
3.0	613500	296393.73	4.69
SLAB 2			
Load (kN)	*Ma* (N-mm)	*Ie* (mm^{4})	Theoretical Deflection (mm)
0.0	0.0	0.0	0.0
1.23	313500	582980.24	1.23
2.37	413500	396390.21	2.37
3.56	513500	327521.60	3.56
4.69	613500	296393.73	4.69
5.77	713500	280318.18	5.77
SLAB 3			
Load (kN)	*Ma* (N-mm)	*Ie* (mm^{4})	Theoretical Deflection (mm)
0.0	0.0	0.0	0.0
1.5	313500	582980.24	1.23
2.0	413500	396390.21	2.37
2.5	513500	327521.60	3.56
3.0	613500	296393.73	4.69
3.5	713500	280318.18	5.77

SLAB 4			
0.0	0.0	0.0	0.0
1.17	247500	924376.56	0.61
2.14	441500	370671.15	2.71

Table 4.10. Theoretical Deflection of Reinforced Concrete One-Way Slab (Continuation)

Load (kN)	*Ma* (N-mm)	*Ie* (mm^4)	Theoretical Deflection (mm)
2.21	455500	360085.45	2.88
2.45	503500	332095.27	3.44
2.71	555500	311709.74	4.05
2.78	569500	307432.63	4.20
3.11	635500	291967.54	4.94
3.42	697500	282293.36	5.60
3.75	763500	275159.76	6.28
4.04	821500	270644.75	6.87
4.25	863500	268091.12	7.29
4.32	877500	267345.65	7.43
4.33	879500	267243.00	7.45
4.35	883500	267040.48	7.49
4.39	891500	266646.26	7.57
SLAB 5			
Load (kN)	*Ma* (N-mm)	*Ie* (mm^4)	Theoretical Deflection (mm)
0.00	0.0	0.0	0.0
1.44	301500	624061.2	1.10
1.92	397500	414504	2.18
2.21	455500	360085.5	2.88
2.29	471500	349477.3	3.07
2.58	529500	320903.5	3.75
2.94	601500	299087.9	4.56
3.03	619500	295123.9	4.76
3.35	683500	284176.6	5.45
3.65	743500	277057.5	6.08
3.95	803500	271907.9	6.69
4.25	863500	268091.1	7.29
4.51	915500	265544.6	7.80
4.66	945500	264320.2	8.09

SLAB 6			
0.00	0.0	0.0	0.0
1.46	305500	609647.44	1.14
1.97	407500	402850.68	2.30
2.18	449500	364460.97	2.80
2.41	495500	336024.70	3.35

Table 4.10. Theoretical Deflection of Reinforced Concrete One-Way Slab (Continuation)

Load (kN)	*Ma* (N-mm)	*Ie* (mm^{40})	Theoretical Deflection (mm)
2.60	533500	319371.18	3.79
2.75	563500	309213.70	4.14
2.93	599500	299558.09	4.54
3.25	663500	287150.25	5.24
3.38	689500	283350.76	5.51
3.48	709500	280795.33	5.73
3.62	737500	277667.57	6.02

Comparisons of Theoretical and Experimental Deflections

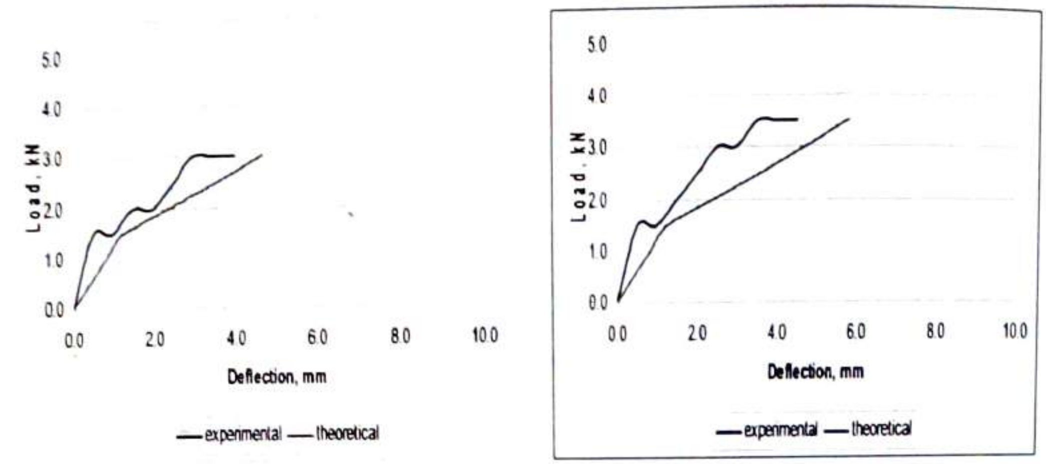
Table 4.11 shows, and Figure 4.7 plots the difference of the theoretical deflections, Δ_T and experimental deflections, Δ_E of reinforced concrete one-way slabs in the study. The difference between the deflections is computed up to the ultimate load capacity of the slabs. This is computed from $(\Delta_T - \Delta_E) / \Delta_T \times 100\%$ and is done by using the Microsoft Excel.

From the results shown in Table 4.11, the experimental deflections are less than the theoretical deflections. The average difference of deflections for different loads are 19.70%, 26.21%, 19.75%, 27.70%, 30.76% and 28.94% for slab 1, 2, 3, 4, 5, and 6, respectively. These differences averaged to 25%. Thus, the experimental deflections are 25% less than the theoretical deflections.

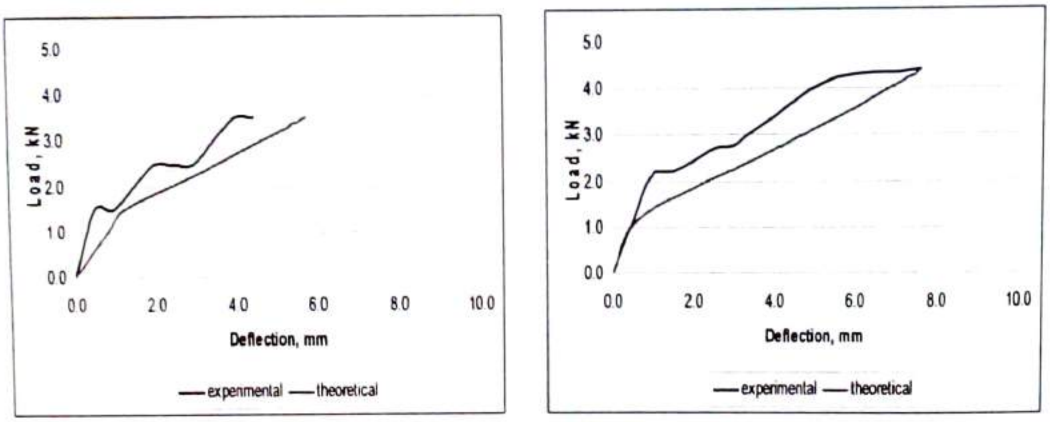
Table 4.11. Theoretical and Experimental Midspan Deflections of Reinforced Concrete One-Way Slab

SLAB 1			
Experimental Load (kN)	Experimental Deflection (mm)	Theoretical Deflection (mm)	% Difference $[(\Delta_T-\Delta_E)/\Delta_T \times 100]$
0.0	0.0	0.0	0.0
1.5	1.0	1.23	18.70
2.0	2.0	2.37	15.61
2.5	2.5	3.56	29.78
3.0	4.0	4.69	14.71
Average			19.70
SLAB 2			
0.0	0.0	0.0	0.0
1.5	1.0	1.23	18.70
2.0	1.5	2.37	36.71
2.5	2.0	3.56	43.82
3.0	3.0	4.69	36.03
3.5	4.5	5.77	22.01
Average			26.21
SLAB 3			
Experimental Load (kN)	Experimental Deflection (mm)	Theoretical Deflection (mm)	% Difference $[(\Delta_T-\Delta_E)/\Delta_T \times 100]$
0.0	0.0	0.0	0.0
1.5	1.0	1.23	18.70
2.0	1.5	2.37	36.71
2.5	3.0	3.56	15.73
3.0	3.5	4.69	25.37
3.5	4.5	5.77	22.01
3.35	4.5	6.08	25.99

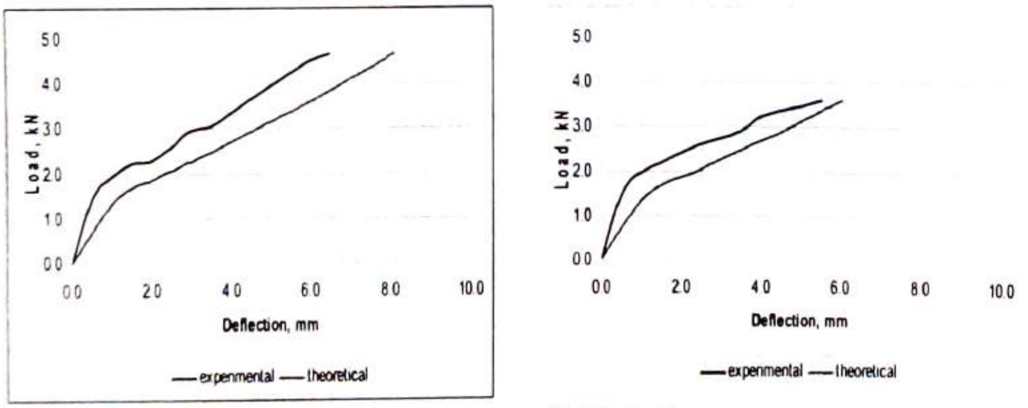
3.65	5.0	6.69	25.26
3.95	5.5	7.29	24.55
4.25	6.0	7.80	23.08
4.51	6.5	8.09	19.65
	Average		30.76
SLAB 6			
Experimental Load (kN)	Experimental Deflection (mm)	Theoretical Deflection (mm)	% Difference $[(\Delta_T-\Delta_E)/\Delta_T \times 100]$
0.00	0.0	0.0	0.00
1.46	0.5	1.14	56.14
1.97	1.0	2.30	56.52
2.18	1.5	2.80	46.43
2.41	2.0	3.35	40.30
2.60	2.5	3.79	34.04
2.75	3.0	4.14	27.54
2.93	3.5	4.54	22.91
3.25	4.0	5.24	23.66
3.38	4.5	5.51	18.33
3.48	5.0	5.73	12.74
3.62	5.5	6.02	8.64
	Average		28.94



(a) Slab 1 (b) Slab 2



(c) Slab 3 (d) Slab 4



(e) Slab 5 (f) Slab 6

Figure 4.7. Experimental and Theoretical Load-Deflection Relationship of Reinforced Concrete One-Way Slab

Theoretical and Experimental Ultimate Load Capacity

Table 4.12 shows the theoretical and experimental ultimate load capacity of the slab specimens.

Table 4.12. Theoretical and Experimental Ultimate Load Capacity of Slab

Specimen	Experimental Ultimate Load Capacity, kN	Theoretical Ultimate Load Capacity, kN
Slab 1		2.60
Slab 2	4.00	
Slab 3	4.00	
Slab 4	4.39	
Slab 5	4.66	
Slab 6	3.62	
Average	4.13	2.60

The experimental ultimate load capacity of slab 1 was not determined since the UTM was mistakenly unloaded before it reaches the peak load.

Reinforcing Steel Bars

Table 4.13 shows the yield strength of each reinforcing steel bar samples. The corresponding elongation is also shown.

Table 4.13. Yield Strength and % Elongation of Reinforcing Steel Bars

Specimen	Yield Strength	% Elongation
1	278.52	6.49
2	218.84	8.17
3	238.73	5.79
Average	245.36	6.82

The average yield strength of the reinforcing steel bars, which is 245.36 MPa, is used as the f_y for the theoretical computations.

Summary of Findings

After conducting the experiment, the following findings are obtained:

1) The concrete used in the study reaches ultimate load at a strain of 0.005 mm/mm.
2) The compressive strength of concrete, f'_c is 20.56 MPa.
3) The experimental modulus of elasticity of concrete, which is obtained from the stress-strain curve, is 4,513.09 MPa while the theoretical modulus of elasticity, which is computed based on f'_c and density of concrete, w_c is 23,686.15 MPa.
4) The average experimental deflection of the slabs is 25% less than the theoretical deflections.
5) The average experimental ultimate load capacity of the slabs is 4.13 kN while theoretical ultimate load capacity is 2.6 kN. Thus, the experimental ultimate load capacity is 37% greater than the theoretical ultimate load capacity.
6) The yield strength of reinforcing steel bars, f_y is 245.36 MPa with an elongation of 6.82%.

Conclusions and Recommendations

Conclusions

The results of the study show that the experimental deflections of the reinforced concrete one-way slabs are significantly less than its theoretical deflections. The average experimental deflection is 25% less than the theoretical deflections.

The study also shows that the experimental ultimate load capacity of the reinforced concrete one-way slabs are significantly greater than the theoretical ultimate load capacity. The average experimental ultimate load capacity is 37% greater than the theoretical predictions.

Recommendations

The following are recommended for future similar studies:

1) Study of the Load-Deflection Characteristics and Ultimate Load Capacity of Reinforced Concrete Beams.
2) Fabrication of a mechanical device that can measure the strain of steel bars.
3) Extensive study that considers a wide variation of concrete strengths, reinforcing steel bars, and slab cross-sections.

APPENDIX

ASTM Standards Methods Used

i. ASTM C 39-90 – Specifications for Sieve Analysis of Fine and Coarse Aggregates
ii. ASTM C 136 – Test Method for Sieve Analysis of Fine and Coarse Aggregates
iii. ASTM C 29 – Unit Weight Determination Test
iv. ASTM C 127 – Test Method for Specific Gravity and Absorption of Fine and Coarse Aggregates
v. ASTM C 143 – Test Method for Slump of Concrete
vi. ASTM C 39-86 – Test Method for Compressive Strength of Concrete
vii. ASTM A 370-92 – Standard Test Methods and Definitions for Mechanical Testing of Steel Products

Bibliography

Everard and Tanner, "Structural Design Considerations," *Reinforced Concrete Design,* 1996, pp. 12-13.

Gillesania, D.I.T., "Analysis and Design of Beams," *Fundamentals of a Reinforced Concrete Design,* 2002, p.24.

Nawy, E., "Deflection Behavior of Beams," *Reinforced Concrete: A Fundamental Approach,* 1996, pp. 275-281.

Nilson, A.H., "Design of One-Way Slabs," *Design of Concrete Structures,* 12th Ed., 1997.

Olonisakan, " Reinforced Concrete Slabs with Partial Lateral Edge Restraint," *http://www-civ.eng.cam.ac.uk/abstract/akinabs.htm.*

Ombres, L., Alkhrdaji T., et. al., "Flexural Analysis of One-Way Concrete Slabs Reinforced with GFRP Rebars," *http://web-test.umr.edu/~utc/pubs/conf/ 2000/oneway.pdf.*

Spiegel, L. and Limbrunner, G., "Deflections," *Reinforced Concrete Design,* 4th Ed., 1998, p. 215-216.

Taplin, G. and Attard, Mario, "Three-point Loading Test on Composite Slabs with Interface Slip," *http://www.civil.eng.monash.edu.au/people/staff/ teaching_staff/taplinfolder/gtpapers/1993_ACMSM.pdf.*