# **CALCULUS IN BIOLOGY**

# Ruth P. Serguiña

C<br>C<br>C e recent developments in biole<br>e which necessitate the applica n biological scient<br>pplication of calf<br>f biological prob<br>art in this decade omplicated prob<br>Calculus is ofte<br>It may be trace ose which necessitate the application of calculus. Calculus is ofte the quantitative analysis of biological problems.<br>Iculus in biology did not start in this decade only. It may be trace ee<br>isto<br>alt f d<br>pro<br>ca<br>cd

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leulus in biology did not start in this decade only. It may be to<br>
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ludy of populations. Stirred by such the ulations. Stirred by such theory, biol<br>hematicians to transform biological prol<br>ogistic equation was used by the lik<br>orld-population growth and Pearl (197 the quantitative analysis of biol<br>lculus in biology did not start in<br> $\frac{1798}{1798}$  when Thomas Malthus pu<br>*lation as it Affects the Future* Id no longer support its population. His theory made a gre<br>le study of populations. Stirred by such theory, biologis<br>is the help of mathematicians to transform biological problem b seek the help of mathematicians to tran<br>thematical problems.

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m a different perspective, far<br>f the practical applications of car<br>require any in-depth knowled com<br>do<br>dow tamples below are some of the practical applications of calculations. elow are some of the practical applications of c<br>problems do not require any in-depth knowle<br>Il show how the basic ideas of calculus, e.g., th<br>on and integration, are used in biology. do not require any in-depth know the basic ideas of calculus, e.g<br>egration, are used in biology. These problems do not require any in-depth knowledge of these will show how the basic ideas of calculus, e.g., the limit fferentiation and integration, are used in biology.<br>Illowing problems and solutions are selected from ulu<br>ted<br>sxa<br>ut<br>infi ese will show how the basic ideas of calculus, e.g., the limit entiation and integration, are used in biology. .g.<br>:he tegration, are used in biol<br>and solutions are selected

 $\left[2\right]$ , Gentry  $\left[2\right]$ , and Cullen  $\left[3\right]$ . ifferentiation and integration, are used in biology.<br>
bllowing problems and solutions are selected from th<br>
Gentry [2], and Cullen [3]. d<br>m<br>l<br>l

**unctional response curve**. Suppose we are studying the a predator (e.g., a fox). How does the number  $y$  of prey ten over a prescribed period of time depend upon the es<br>e.g<br>pi u<br>i e stu habits of a predator (e.g., a fox). How does the number y of prey<br>abbits) eaten over a prescribed period of time depend upon the<br> $169$ predator (e.g., a fox). How does the number y of pre-<br>prover a prescribed period of time depend upon the

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of the prey? Surely as x increases, that is, as the undant,  $y = f(x)$  increases. Since the predator can contain the case that  $f(x) \approx f(x)$ the prey? Surely as x increases, that is, as the prey becan  $v = f(x)$  increases. Since the predator can consume of ha<br>eda<br>th oundant,  $y = f(x)$  increases. Since the predator can<br>number of prey, it should be the case that  $f(x_1)$ <br>of  $x_1$  and  $x_2$ . The curve relating y and x should pos<br>ote In 1959 Hollings discovered a rational function in<br>e<br>re nl<br>la<br><sup>Ol</sup> umber of prey, it should be  $x_1$  and  $x_2$ . The curve relating<br>e. In 1959 Hollings discovered brey, it should be the case that  $f(x_1) \approx f(x_2)$  for large<br>The curve relating v and x should possess a horizont f  $x_1$  and  $x_2$ . The curve relating y and x should possess a hot<br>te. In 1959 Hollings discovered a rational function that wor<br>bing the feeding habits of invertebrate predators and some t n 1<br>c  $\frac{1}{2}$ ollings discovered a rational function that works well<br>
ing habits of invertebrate predators and some fish:<br>
<sup>1</sup> escribing the feeding habits of invertebrate predators and some fish<br>  $y = \frac{ax}{1 + bx}$ , for  $x \ge 0$ . inverted<br>1

$$
y = \frac{ax}{1 + abx}, \text{ for } x \ge 0.
$$

imit as<br>to be<br> $\frac{1}{2}$ . is x tends to positive infinity, we find the

$$
\text{con-tal asymptote } t
$$
\n
$$
\lim_{x \to +\infty} \frac{ax}{1 + abx} = \frac{1}{b}
$$

 $\frac{1}{1}$ he line  $y = \frac{1}{b}$  is a hor

iyi<br>*n*o es<sub>,</sub> urve, called *Hollings' functional response curve*, is show<br>estable es<sub>.</sub> below:





he be<br>rug i is neutralized at an exponential rate<br>of the drug is neutralized before the<br>by  $A(t)$  the amount of drug in a<br>hen eutralized at an ex ี่ a<br>eเ<br>ทเ  $\frac{1}{10}$  is a set of a drug in the body. A drug is adm esidues of a drug in the body. A drug is administered every 4<br>oses of 2 mg.. The drug is neutralized at an exponential rate<br>onstant  $k = -0.5$ . Not all of the drug is neutralized before the el<br>d<br>re te constant  $k = -0.5$ . Not all of the drug is neutralized before the ose is administered. Denote by  $A(t)$  the amount of drug in a  $=$ <br>is m ef<br>ru lose is administered. Denote by  $A(t)$  the amount of drug in<br>  $\lambda$ 's system at time t (in hours). Then  $\frac{1}{2}$ ystem at time *t* (in hour  $A_0 e^{-0.5t}$ .  $\int$ n doses of<br>e constant

$$
A(t)=A_0 e^{-0.5t}.
$$

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RUTH P. SERQUIÑA<br>termine the drug leftover from the first dose, we contract the drug leftover from the first dose, we contract the drug leftover from the first dose, we contract the drug leftover from the first dose, we co

etermine the drug leftover from  
\n
$$
\lim_{t \to 4^{-}} A(t) = \lim_{t \to 4^{-}} 2e^{-0.5t} = \frac{2}{e^{2}}.
$$

the drug leftover from the first dose is  $2/e^2$ . To compute for the the text time interval [4,8), the drug leftover in the first dose cluded. Hence for the given intervals, we have the  $g(A(t))$ . the up the the dose is  $2/e^2$ . To compute the interval  $[1, 8)$  the drug leftover in the f We the next time interval [4,8), the drug leftover in the first dose<br>be included. Hence for the given intervals, we have the<br>ponding  $A(t)$ : Hence for the given intervals, we have the

responding 
$$
A(t)
$$
:

\nfor [0,4):  $A = 2e^{-0.5t}$ , the leftover is  $2/e^2$ ;

\nfor [4,8):  $A = (2+2/e^2)e^{-0.5t}$ , the leftover is  $2/e^2 + 2/e^4$ ;

\nfor [8,12):  $A = (2 + 2/e^2 + 2/e^4)e^{-0.5t}$ , the leftover is  $2/e^2 + 2/e^4 + 2/e^6$ .

\ngeneral, for any time  $t \in [4n,4(n+1)]$ ,  $A$  is given by the formula

\n $A_n(t) = (2 + R_{n-1})e^{-0.5t}$ ,

$$
A_{n}(t)=(2+R_{n-1})e^{-0.5}t,
$$

+  $R_{n-1}$ ) $e^{-0.5t}$ ,<br>the leftover of the precision-<br>will determine the resi-

 $n_{-1}$  is the leftover of the preceding time interval.<br>  $n_{\lambda}$ , we will determine the residual amount of drug in the patit the end of the *n*th dosage period. e<br>ag<br>le mount of<br>the drug. nterval.<br>
of drug in The *n*th dose

show the *n*th dosage period.<br>
desidue is simply the leftover of the drug. Hence, for the first<br>
t is.  $n = 1$ , the residue is  $R_1 = 2/e^2$  (this is the leftover after the simply the leftover of the drug. Hence, for the 1, the residue is  $R_1 = 2/e^2$  (this is the leftover after  $n = 1$ , the residue is  $R_1 = 2/e^2$  (this is the leftover after the

$$
n = 2, R_2 = 2/e^2 + 2/e^4;
$$
  
\n
$$
n = 3, R_3 = 2/e^2 + 2/e^4 + 2/e^6;
$$
  
\n
$$
n = 4, R_4 = 2/e^2 + 2/e^4 + 2/e^6 + 2/e^8.
$$

meral, the residue for the *n*th inter is  $\frac{1}{2}$ 

$$
R_n = \lim_{t \to 4^-} A_n = \sum_{i=0}^n 2e^{-2i}.
$$

The Michaelis-Menten relation. Special proteins known as *enzymes* act as catalysts for a wide variety of chemical reactions in living things. The term *substrate* refers to the substance that is being acted upon. In 1913, Mihealis and Menten devised the formula (see below) relating the initial speed  $V$  with which the reaction begins to the original amount of sub-strate  $x$ :

$$
V = \frac{ax}{x+b}
$$

(Typical units are moles/liter for x and moles/liter/second for  $V$ .) This equation has been verified experimentally for a variety of enzyme controlled reactions. There also exist theoretical derivations of the equations. When x is very large,  $V \approx a$ . We can show this by taking the limit of  $V$  as  $x$  tends to infinity:

$$
\lim_{x \to +\infty} \frac{ax}{x+b} = a.
$$

Thus, the line  $V = a$  is a horizontal asymptote.

Bites from a poisonous snake. If a person bitten by a poisonous Snake receives an immediate shot of antivenum, then t seconds after the shot is given there will be

$$
y = \frac{0.5t + 2000}{2t + 3}
$$
 ppm

of poison in the victim's blood.

To find the concentration of poison in the blood as time passes by, we take the limit of  $y$  at infinity, concentration of poison in the blood decreases toward 0.25 ppm as time goes by.  $\lim y = 1/4$ . Eventually, the

### **RUTH P. SEROUIÑA**

RUTH P. SERQUIÑA<br> **Stantaneous rate of growth of a tumor.** A tumor is esti itati<br>: 1<br>`he The instantaneous rate of growth of a tumor. A tumor is estimated<br>twe a total mass of  $6 \times 10^{-3} t^2$  gm., *t* days after its discovery. How fase<br>tumor growing on the 8th day? e tumor growing on the 8th day'<br>Let  $f(t) = 6 \times 10^{-3}t^2$ . Then the is

otal mass of  $6 \times 10^{-3} t^2$  gm., *t* days after its discovery. How fast<br>or growing on the 8th day?<br> $t = 6 \times 10^{-3} t^2$ . Then the instantaneous rate of growth on the 8th te  $e^{i\theta}$ day is

$$
f'(8) = 12 \times 10^{-3}(8) = 96 \times 10^{-3} \text{ gm./day.}
$$

 $2 \times 10^{-3}$ (8) = 96×10<sup>-3</sup> gm./day.<br>**res.** The number of forest fires in a particular region can<br>function of the number of days x since the last measura<br>rever, the fires fall into two categories:  $\begin{array}{c} \n \text{on a} \\
 x \text{ s} \\
 \text{or} \\
 \end{array}$ umber of forest fires in a particular region can be<br>f the number of days x since the last measurable In the number of the number of the number of the fires fall into two controls. unction of the number of days  $x$  since the last me ver, the fires fall into two categories: However, the fires fall into two cate<br>ose caused by nature (i.e., lightning

ose caused by nature (i.e., lightning), and<br>ose attributable to man.

nose attributable to ma<br> at<br>Literatura<br>Literatura

Let

 $\frac{1}{2}$ 

the number of fires due to natural cause<br>the number of fires caused by man,

the number of fires due to natural<br>the number of fires caused by man<br> $N(x) + M(x)$ , the total number of fi

 $N(x) + M(x)$ , the total number of fires, and

 $N(x) + M(x)$ , the total number of fires, and<br>  $M(x)/F(x)$ , the proportion of fires that are caused by man-<br>
change of the relative number of man-related fires is

te of change of the relative number of mar<br>  $f(x) = D_x[M(x)/F(x)].$ es<br>and the contract of the con

 $D_{\rm x}[M(x)]$ 

 $\int f(x) \, dx$ 

e substitute 
$$
M'(x) + N'(x)
$$
 for  $F'(x)$ , then  
\n
$$
R'(x) = \frac{N(x)M'(x) - M(x)N'(x)}{(M(x) + N(x))^2}.
$$
\nIf  $N(x) = (0, 1)(x - 1) M(x) = (0, 4)x^2$ 

 $N(x) = (0.1)(x - 1)$ ,  $M(x) = (0.4)x^2$  and  $F(x) = 0.4x^2 + 0.1x - 0.1$ <br>ate of change in the proportion of man-related fires is 1),  $M(x) = (0.4)x^2$  and<br>the proportion of man e rate of change in the proportion of mar<br>173

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$$
R'(x) = \frac{(0.04)(x^2 - 2x)}{(0.4x^2 + 0.1x - 0.1)^2}.
$$

(1.04)<br>(1.04)<br>(1.04) **Letabolism.** To test for diabetes, a patient is s<br>of sugar. The amount of glucose in the patient<br>over an interval [0.7]. If the amount of glucos o<br>eal<br>| c st for diabetes, a patient is sub patient is subjected to a<br>in the patient's urine is<br>not of glucose is given by<br>what rate is the patient uantity of sugar. The amount of glucose in the patient's urine is<br>easured over an interval [0,*T*]. If the amount of glucose is given by<br> $10 - 0.6t^2$  where t denotes hours, at what rate is the patient quantity of sugar. The amount of glucose in the patient's urine is<br>measured over an interval [0,*T*]. If the amount of glucose is given by<br> $= 10 - 0.6t^2$ , where *t* denotes hours, at what rate is the patient<br>volizing the ugar. The amount of glucose in the pation of  $\frac{1}{2}$  $\sum_{i=1}^{n}$ lizing the sugar two hours after the test begins? Since  $g'$ <br>e patient is metabolizing at a decreasing rate of  $-2.4$  unit<br>e size of the human eve pupil. The size of a human e gar two hours after the test begins? Since  $g'(2) = -2$ <br>netabolizing at a decreasing rate of  $-2.4$  units/hour. is

ient is metabolizing at a decreasing rate of  $-2.4$  units/hour<br> **ze of the human eye pupil.** The size of a human eye pu<br>
the amount of light incident to the retina of the eye **ye pupil.** The size of a human eye pupil is<br>it incident to the retina of the eye. The<br>nship is the amount of light incident to the retina of the eye. 1<br>describing this relationship is relationship is

ution describing this relative to the following equation:

\n
$$
A(l) = \frac{40l^{-0.4} + 23.7}{l^{-0.4} + 3.95},
$$

is the area of the pupil and  $l$  is the quantity<br>er unit of time incident on the retina of the eye<br>rate of change in the pupil area corresponding the area of the pupil and  $l$  is the quantity of visi-<br>unit of time incident on the retina of the eye

The area of the pupil and  $l$  is the quantity of viss unit of time incident on the retina of the eye.<br>te of change in the pupil area corresponding to a changiven by  $\overline{\phantom{a}}$ er unit of time incident on the retina of the eye.<br>rate of change in the pupil area corresponding to a<br>is given by given by<br> $-53.72l^{-1.4}$ 

$$
A'(l) = \frac{-53.72l^{-1.4}}{(l^{-0.4} + 3.95)^2}.
$$

ite model. *Parasites* are animals or organisms that live on or reganism called *host*. Parasites can either be helpful or eir host. (Ruminant animals such as sheeps are dependent on complete their digestive process). Para ither be help **arasite model. Parasites** are animals or organisms that live on or er organism called *host*. Parasites can either be helpful or animals or c<br>Parasites can<br>als such as to their host. (Ruminant animals such as sheeps are deper<br>to complete their digestive process). Parasites are fre<br>to biologically control pests. One such parasite desti mimals such as sheeps are dependent on<br>tive process). Parasites are frequently<br>pests. One such parasite destroys the<br>spiders in an area is  $H$  and the relative so complete their digestive process). Parasites are<br>to biologically control pests. One such parasite of<br>pider. If the number of spiders in an area is  $H$  and<br>parasites is  $P$ , then the number  $H$  is a function of  $P$ : by yed to biologically control pests. One such parasite destroys the f a spider. If the number of spiders in an area is  $H$  and the relative r of parasites is  $P$ , then the number  $H$  is a function of  $P$ : If the number of spiders in a<br>ss is  $P$ , then the number  $H$  is arrasites is *P*, then the number *H* is a function of *P*:<br> $M(1 - 2P<sup>3</sup>)$ , in another organism called *host*.

Physical is  $P$ <br>=  $M(1 - 2P<sup>3</sup>)$ ,

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RUTH P. SERQUIÑA<br>the maximum host population. However, this part *I* is the maximum host population. However, this parasite can produce when the temperature is between 24 and 30  $^{0}C$ .<br>tently, the relative number of parasites is a function of the ture *t*. Assume that reproduce when the temperature is between 24 quently, the relative number of parasites is a function rature  $t$ . Assume that the relative number of parasites is a function of the ssume that<br>  $(30-t)/9$ , ssume that

 $\frac{A}{2}$ 

 $(-24)(30 - t)/9$ ,<br>although the spider population is not sensitive to the<br>tion size *H* is affected by the temperature. This can be des temperature

on<br>fol<br>. is population size  $H$  is affected by the temperature. This can be des ize *H* is affected by t<br>wing composition equ<br> $t = H(P(t))$  for  $t \in \mathbb{R}$ Th<br>. the following composition equals<br>  $f(t) = H(P(t))$ , for  $t \in [2e^{\theta} \times 1e^{-\theta}]$ 

$$
H(t) = H \circ P(t) = H(P(t)), \text{ for } t \in [24,30].
$$

 $H \circ P(t) = H(P(t)),$  for  $t \in [24,30].$ <br>
e temperature is 28<sup>°</sup>C, is the spider point is 28  $^{\circ}C$ , is the spider population increasing or rate? To answer this, we need to evaluate the 28.<br>28.  $\therefore$ nd<br>'(2<br>pos t what rate? To answer this, we need to evaluate the at  $t = 28$ .<br>on of functions ni<br>T  $= 28$ 

d<br>derivative<br>derivative

ratio 
$$
H'(28)
$$
 at  $t = 28$ .  
By composition of functions,  
 $\hat{H}(t) = M(1-2[(t-24)(30-t)/9]^3)$ .

the derivative of 
$$
\hat{H}(t)
$$
 is  
\n $\hat{H}'(t) = M(-2/9^3)(3)[(t-24)(30-t)]^2 [-2t+54].$ 

 $M(-2/9^3)(3)[(t-24)(30-t)]^2$ <br>ation above, we have  $\hat{H}'(28)$  =<br>means that the spider populat e equation above, we have  $H'(28) = (256/243)M \approx 1.053M$ , whom This means that the spider population is increasing at the rate spiders per <sup>0</sup>C. uation above, we have  $H'(28) = (25$ <br>s means that the spider population  $\frac{1}{g}$ This means that the spider population is increasing at the rate of piders per  ${}^{0}C$ . 1.053M spiders per  ${}^{0}C$ .

er <sup>0</sup><br>L**av**<br>osit:<br>sse<br>el. ne of the contributing factors in the acc<br>side the blood vessels is the fact that the actors in tl<br>s the fact<br>n the flow<br>te near the pid (fat) deposits inside the blood vessels is the fact that the flow of d near the vessel walls is much slower than the flow of blood at the fat) deposits inside the blood vessels is the fact that the flow of<br>
in the vessel walls is much slower than the flow of blood at the<br>
the vessel. Due to the slower flow rate near the walls, the linic ear the vessel walls is much slower than the flow of blood at the vessel. Due to the slower flow rate near the walls, the lip If the vessel. Due to the slower flow rate near the walls, the lipides have a greater chance of becoming attached to the vessel walls by vertice near the walls, the sing attached to the vessel v uv<br>ll: left use of becoming attached to the vessel wall<br>ads to a heart attack. les have a greater chance of b<br>
entually leads to a heart attack<br>
<sup>1</sup>



Figure 2

French physician Poiseuille discovered the relationship that<br>he velocity  $V$  of blood flow as a function of the distance,  $r$ , from<br>of a blood vessel. It is now known as Poiseuille's Law: hysician Pois<br>ty *V* of blood<br>d vessel. It is r locity V of blood flow as a function of the distance, r, from<br>lood vessel. It is now known as Poiseuille's Law: enter of a blood vessel. It is now known as Pointer of a blood vessel. It is now known as Pointer e<br>E

$$
V = \frac{\rho}{4\lambda\eta} (R^2 - r^2),
$$

where

locity of blood flow<br>Iius of the blood ve

 $\frac{1}{2}$  from ood vess<br>the santa

 $\theta$  distance from the cen From the center,<br> $\lambda$  and n are n

e numbers  $\rho$ ,  $\lambda$ , and  $\eta$  are physical con<br>re, length, and viscosity.<br>he outer radius of a blood vessel can be ch  $\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$ or<br>y

to<br>ug<br>el<br>al be changed by administeries<br>
the radius of a blood vessel can be changed by administeries<br>
changed by administeries<br>
changed the vessel (R decreases) or dilate the vessels.<br>
Aspiring dilates blood vessels. Assume that an nd visco<br>ius of a<br>r const the radius of a blood vessel can be channel.<br>  $\frac{d}{dx}$  decreases the correct  $\frac{d}{dx}$  decreases which either constrict the vessel ( $R$  decreases) or d asses). Aspirin dilates blood vessels. Assume that aspiring on a doctor's orders" and as a result, the r  $R$  accession. Aspiring dilates blood vessels. Assume that an independent of  $\mathbb{R}^n$ Aspirin dilates blood vessels. Assume that an individual on a doctor's orders" and as a result, the radius  $R$  of her ries increased in size at a rate of k 2 aspirins on a doctor's orders" and as a re s blood arteries increased in size at a rate of si

 $= 2 \times 10^{-4}$  ci

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At what rate would the velocity of the blood flow be changing? The answer is obtained by computing the derivative of  $V$  using the Chain Rule:

$$
\frac{dV}{dt} = \frac{dV}{dR} \frac{dR}{dt}
$$

Since  $\frac{dV}{d\Omega}$ dR  $(4\lambda \eta)$ R, the rate of change in  $V$  is

$$
\frac{dV}{dt} = 2\left(\frac{\rho}{4\lambda\eta}\right)R(2\times10^{-4})\text{cm./min..}
$$

If  $R = 0.02$  cm. and  $\left(\frac{\rho}{42\pi}\right) = 1$ , then Ιλη∠ the rate of change of the velocity does not depend on the distance  $r$  from the center of the vessel. dV  $\frac{dv}{dt}$  = 8×10<sup>-6</sup> cm./min.. Note that

Gazelle population. The size of a gazelle herd is a function of the amount of the edible grasses within its grazing territory. The amount of grass is estimated by sampling techniques to be  $x$  tons. The size of the gazelle herd is assumed to be

$$
g(x) = \begin{cases} 0, & \text{if } x < m, \\ (x-m)^2 - (x-m) + 2, & \text{if } x \ge m, \end{cases}
$$

where  $m$  is the minimum amount of grass necessary to sustain a pair of gazelles.

But the amount x of grass is a function of the total rainfall  $r$  over the grazing region. Assume that

$$
x(r) = 40r - r^2.
$$

Then the size of the gazelle herd is a function of the rainfall

$$
\hat{g}(r) = (g \circ x)(r) = g(x(r)).
$$

The formula for the gazelle population as a function of rainfall is

$$
\hat{g}(r) = \begin{cases}\n0, & \text{if } 40r - r^2 < m \\
(40r - r^2 - m)^2 - (40r - r^2 - m) + 2, & \text{if } 40r - r^2 \ge m\n\end{cases}
$$

If  $m = 10$ , the rate of change of the herd size due to a change in rainfall is

$$
\frac{d\hat{g}(r)}{dr} = 2(40r - r^2 - 10)(40 - 2r) - 40 + 2r.
$$

Basal metabolism. This term is used to describe the normal chemical activity in an organism not subject to stress - for instance, a plant growing under ideal conditions or an animal resting over a period of time.

The metabolic rate of an animal will vary in response to environmental changes (temperature, humidity, air quality) and changes in physical activity. As air temperature fluctuates on a daily basis, the **basal** *metabolic rate* (BMR) of an animal will vary over a *diurnal cycle*; the BMR increases at night to compensate for the lower temperature and decreases during the day

Metabolic rates are expressed in several equivalent ways - as a measure of heat produced in kilocalories per unit (kcal/hr), as a measure of oxygen consumption per unit of body weight (cm.<sup>3</sup>  $O_2/g$ .), and as a measure of carbon dioxide expelled per unit of time (cm.  $^{3}$  CO<sub>2</sub>/hr.). In all cases, the total basal metabolism BM over a period is obtained as the integral of the basal metabolic rate over the time interval

$$
BM = \int_{t_1}^{t_2} BMR(t) dt.
$$

Suppose the BMR is given by the following function:

#### RUTH P. SEROUIÑA

$$
BMR(t) = -(0.15) \cos\left(\frac{\pi t}{24}\right) + 0.3 \text{ kcal/hr.}
$$
  
sequently, the BM value for a one-day peri-

RUTH P. SEROUINA +0.3 kcal./hr.. M value for a one-day period would be<br>  $\left(\pi t\right)$ 

sequently, the BM value for a one-day period would b  
\n
$$
BM = \int_0^{24} \left[ -(0.15) \cos\left(\frac{\pi t}{24}\right) + 0.3 \right] dt = 7.2 \text{ kcal./day.}
$$

rould correspond to the BM of a mouse, whereas the value for an<br>uman would be approximately 2000 kcal./day. orrespond to t<br>vould be appro rh<br>ay uman would be approximately 2000 kcal

00<br>rdi<br>as oximately 2000 kcal./day.<br>
easuring cardiac output, one me Cardiac output. In measuring cardiac output, one method, known as *ve-dilution method*, is performed as follows. A fixed amount of a injected into a vein or the right side of the heart. This dye then is easuring cardiac output, one method, known a<br>s performed as follows. A fixed amount of<br>or the right side of the heart. This dye then i<br>irough the heart to the lungs, back to the heart nto the arterial system. At a per eral<br>period<br>period the blood is<br>seconds from **net**<br>b a<br>ria is injected into a vein or the right side of the heart. This dye then ideal with the blood through the heart to the lungs, back to the heart nto a vein or the right side of the heart. This dye then is<br>the blood through the heart to the lungs, back to the heart,<br>terial system. At a peripheral artery, the blood is<br>mitored for the presence of the due for 20 ith<br>e<br>in the heart to the lungs, back to<br>
At a peripheral artery, the<br>
the presence of the dye for 30 sec non<br>jec<br>f<br>st by the presence of the dye for 30 seconds from<br>the concentration of dye passing the monitore<br>function,  $c(t)$ , of time. (After about 15 second me of injection. The concentration of dye passing the most<br>is then plotted as a function  $c(t)$  of time. (After about 15 sq The concentration of dye passing the<br>a function,  $c(t)$ , of time. (After about 1<br>e occurs and care in monitoring of<br>regised). The existence surface is defined  $c$ ne<br>in<br>so then plotted as a function,  $c(t)$ , of time. (After about 15 station of the dye occurs and care in monitoring of the on must be exercised.) The cardiac output is defined to is used in the direction of the direction of the last rf<br>st<br>inc n<br>e of the blo<br>
the to be<br>
the ratio of<br>
the over iu:<br>oc<br>ric e exercised.) The cardiac output is defined to be the In must be exercised.) The cardiac output is defined to be f blood pumped per minute. This is obtained as the ratio of f dye injected to the average concentration monitored over l period, multiplied by two, so that it corr utput is defined to<br>obtained as the ratio lood pumped per minute. This is obtained as the ratio of the<br>ve injected to the average concentration monitored assessed.  $\frac{f}{f}$ o the average concentration monitored over the<br>ied by two, so that it corresponds to one minute: ro:<br>ds ve<br>nir ystem.  $\frac{1}{2}$  for the process

second period, multiplied by two, so that it corresponds to one minute:

\n
$$
\text{Cardiac output} = \frac{2[M_g \text{ of injected dye}]}{\frac{1}{30} \int_0^{30} c(t) dt}.
$$
\n(8.1)

the ve-dilution method is used in experiments in basic phy value of the integral,<br>i. The integral,<br> $y/dt$ .

$$
\int_0^{30} c(t) dt,
$$

# THE MINDANAO FORUM

ug<br>no y<br>nt stimated by drawing a co a continuous curve the<br>
on standard graph p<br>
en approximated by co ur<br>gr<br>d<br>20 re plotted on standard graph paper over<br>tegral is then approximated by counting there the curve. This corresponds to interpret integral is then approximated by counting the squares of t<br>nder the curve. This corresponds to interpreting the integr<br> y ds<br>5 1<br>5 1 is interpreting the  $\frac{1}{2}$ <br>g. of dye was injected as an area. e<br>e

nut in an experiment in which 5 mg. of dye was injected at<br>concentration curve was found to be<br>f  $0 \le t \le 3$  or  $18 \le t \le 30$ , and  $= 0$ , the concentration curve was found to be

Let 
$$
t = 0
$$
, the concentration curve was found to be  
\n $c(t) = 0$ , if  $0 \le t \le 3$  or  $18 \le t \le 30$ , and  
\n $c(t) = (t^3 - 40t^2 + 453t - 1026)10^{-3}$ , if  $3 \le t \le 18$ .

if<br>iss<br>'dy .<br>.<br>.rv at  $c(t)$  is in mg./100 ml.. No dye passes the observants and then a large quantity of dye passes ng./100 ml.. No dye passes the observation artery for<br>then a large quantity of dye passes. After 18 seconds,<br>ats of dye has passed. To compute the cardiac output rte<br>ser<br>c seconds, and then a large quantity of dye passes. After 18 seconds easurable amounts of dye has passed. To compute the cardiac out mined by this experiment, we evaluate the average nd<br>is ີ c<br>ne mounts of dye has passed. To com<br>is experiment, we evaluate the aver<br> $(t)dt$ .

rmined by this ex  

$$
A = \frac{1}{30} \int_0^{30} c(t) dt.
$$

Since

$$
\int_0^3 c(t)dt = 0
$$

and

$$
\int_{18}^{30} c(t)dt = 0,
$$

 $\frac{1}{2}$ erage *A* is given by

average A is given by  
\n
$$
A = \frac{10^{-3}}{30} \int_3^{18} [t^3 - 40t^2 + 453t - 1026] dt.
$$
\nstituting the value of A in equation (8.1)

ituting the value of  $A$  in equation (8.1), we obtail<br>is experiment, which is approximately 6.275 liters<br>180 Ilue of *A* in equation (8.1), we obtain the cardiac outp the state of *A* in equation (8.1), we obtain the cardiac outp the state of  $A$  in equation (8.275 liters/min...  $27<sup>2</sup>$ 

# **RUTH P. SERQUIÑA**

MSU-IIT MSU- |!T

elo: uring abundance in the sea. Shown below (Figure 3) is a way typical 1×1 meter square column extending from the ocention. abundan<br>Ba **te sea.** Shown below (Figure 3) is a way the oce  $\overline{a}$ 



A water colu

**Figure 3.** A water column<br>impossible to measure directly the total amount of, say.<br>olumn By taking water sensed and directly the total amount of, say. It is impossible to measure directly the total amount of, say, sar<br>le column. By taking water samples, however, we can measure<br>entration or density at various depths. Let  $f(x) =$  density(in no./n<br>1x. We may imagine that th ir<br>Si f,<br>un<br>y( B:<br>de<br>uy i<br>f t<br>mb y taking water samples, however, we can may<br>ensity at various depths. Let  $f(x) =$  density(in magine that the water column consists of an column. By taking water samples, however, we can measure the tration or density at various depths. Let  $f(x) =$  density(in no./m.<sup>3</sup>) at . We may imagine that the water column consists of an extremely umber of their square h<br>a<br>l E. We may imagine that the water column consists of an extrement of their square layers, each of height  $dx$ , piled on : The number of organisms in a typical layer is of their square layers, each of he<br>
unber of organisms in a typical laye<br>  $(3x)^3$   $\times$  1<sup>2</sup> dx (m.<sup>3</sup>) = f(x) dx. on he number of organisms in a<br>  $\omega/m^3$   $\gg 1^2 dx$  (m.<sup>3</sup>) = f(x) dx.  $\frac{1}{2}$ 

 $(\text{m.}^3) \times 1^2 dx (\text{m.}^3) = f(x)$ 

(x) (no./m.<sup>3</sup>)×1<sup>2</sup> dx (m.<sup>3</sup>) = f(x) dx.<br>
The integral adds all those term from  $x = 0$  to  $x = 20$ <br>  $(x)dx$ , the total number of organisms in the water column<br>
suppose that the density of sardines (no. of fish/m.<sup>3</sup>) is gi  $\frac{1}{2}$ the term from  $x = 0$  to  $x = 200$  to ob

by nat the density of sardines (no. of fish/m.<sup>3</sup>) is given by<br>  $005x(75 - x)$ , where  $0 \le x \le 75$ .

$$
f(x) = .005x(75 - x)
$$
, where  $0 \le x \le 75$ .

 $0.005x(75 - x)$ , where  $0 \le x \le 75$ .<br>True the total number of sardines undetermine the total number of sardines in the water column by thing<br>
ting<br>  $181$ determine

$$
\int_0^{75} 0.005x(75 - x)dx = 351.56.
$$

 $(x - x)dx = 351.56.$ <br>tal number of sardines in the water column is abo ence, the total number of sardines in the water column is about 352.<br>
ensity of the sardines is the largest at the depth 37.5 m.. This can be<br>
v noting that  $f(0) = f(75) = 0$  and  $f'(x) = 0$  when  $x = 37.5$  m.. ty of the sardines is the largest at the depth 37.5 noting that  $f(0) = f(75) = 0$  and  $f'(x) = 0$  when  $x = 3$  ose that a fisherman is trawling sardines. His net

noting that  $f(0) = f(75) = 0$  and  $f'(x) = 0$  when  $x = 37.5$  m..<br>ppose that a fisherman is trawling sardines. His net has an ope uppose that a fisherman is trawling sardines. His net has<br>10 m. wide and 10 m. deep. It is lowered down betw<br>nd 42.5 m. in an attempt to capture the most fish. If ep<br>orr<br>in 10 m. wide and 10 m. deep. It is lowered dow<br>and 42.5 m. in an attempt to capture the most fis<br>ig speed is 20 m./min., how many sardines could in an attempt to capture the most fish. If the nor s 20 m./min., how many sardines could be caught in 15 speed is 20 m./min., how many sardines cou<br>ween the depths  $32.5$  and  $42.5$  m. in a 1 m.<sup>2</sup>  $min.$ ?

he depths 32.5 and 42.5 m. in a 1 m.<sup>2</sup> water column, the mes is

For the values, we get:

\n
$$
\int_{32.5}^{42.5} 0.005x(75 - x) \, dx \approx 69.9 \, .
$$

 $\int$   $\frac{dx}{y}$  and  $\frac{dy}{dx}$  $25x(75 - x)dx \approx 69.9$ .<br>
f the net moves through 1 m., it can capture  $10(69.9) = 69$ <br>
er a 15 minute period, the net is moved through  $20(15) = 39$ <br>
in theory, it would contact  $300(699) = 209,700$  sarding e net moves through 1 m., it can capture  $10(69.9) = 69$ <br>15 minute period, the net is moved through  $20(15) = 30$ nes. Over a 15 minute period, the net is moved through  $20(15) = 3$ <br>Hence, in theory, it would contact  $300(699) = 209,700$  sardine<br>ever, since the escape rate is probably quite high, we would expect inute period, the net is moved through  $20(15) = 300$ If theory, it would contact  $300(699) = 209,700$  sardines.<br>
the escape rate is probably quite high, we would expect to small percentage of this number.  $m$ .. nce the escape rate is probably quite<br>
a small percentage of this number.

nly a small percentage of this number.<br>
examples presented here shows the imperresented here shows the importance of calculus in biology.

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