RECREATIONAL MATHEMATICS
(JUAN TAMAD IN THE WORLD OF MATHEMATICS)

Marcelo M. Alquiza

This is part of a series of accounts about Juan Tamad's adventures in the world of mathematics. What he lacks in menial endeavors, he compensates in productive fantasies, prolific thinking and mental prowess.

The many long hours he indulged himself amidst the quiet meadow under the shady guava tree alongside the cool crystal clear flowing stream were not spent just for nothing. With his back flat on the green grass, one leg akimbo on the other knee, and having his hands under his head as pillows, and his eyes focus yonder into the clear blue sky, Juan has been formulating "plans and strategies" in his head in preparation for meeting challenges of life in the real world. He is thinking, and he is doing mathematics. Juan Tamad is very industrious and very productive in his mind.

The nature of mathematics. The basic nature of mathematics is creative and inventive. Everything in mathematics is man-made, not discovered, in contrast to many other physical sciences. The things in mathematics are the creations of the human minds - ideas and concepts, which are produced by analytical and critical thinking and reasoning. Whether we like it or not, or believe it or not, mathematics is *abstract*, for it originates from the realm of ideas and rich imaginations.

Anybody can create and invent things, concrete or abstract, but not all are desirables and will stay. The acceptability and the power of a created thing is in its utility and applicability into the real world of living. Many inventions and creations in mathematics were done at the outset without any regards to its applications and usefulness. Amazingly, most of them have been found powerful and have rich applications in the physical world.

The power and beauty of calculus. Calculus is one creation in mathematics which is so powerful and has very abundant applications. The invention of calculus toward the end of the seventeenth century opened the floodgates of astonishingly successful disposal of hosts of problems which had been baffling and quite unassailable in earlier days. The beauty of calculus continuously leaves its imprints in all areas and

fields of studies and applications in mankind's endeavors to conquer into new heights and ever broadening horizons of knowledge of the things around him.

The created and invented ideas and concepts in calculus, such as limits, continuity, derivative, etc., can be well understood when put into applications in problem solving. A word of *caution* is well in order at this point - not to rush into some examples for there are almost infinitely many of them.

A note on the learning process. The learning process can be well enhanced when students are recreating and reinventing the ideas and concepts in calculus through applications in problem solving in the real world. It will work best when students are being guided in making-up their own problems as applications to the concepts they are learning. *Sometimes, the problem in problem-solving is in the problem itself.* Students are not involved and motivated in doing the problem which has no appeal and relevance to their real lives. Students will learn most when they are well motivated and deeply involved in what they are doing.

Some demonstrations. Juan Tamad's perspective and persuasion in life is best suited in mathematics. Juan Tamad, just like most mathematicians, is "lazy", but will always *think* and *find* ways of doing things. We will look at two episodes in the life of Juan Tamad in the world of mathematics.

In this episode Juan Tamad goes hi-tech. He is playing games in the video arcade. He has to put into actions what he had been formulating in his head "plans and strategies" for assaults at the "jungle machines" for "war and fun".

This particular afternoon's saga is a "frolic" encounter with a series of *double-or-nothing* game galleries at the arcade. To play, one buys a token and insert at the entrance of each booth which accommodate only one person at a time. Once inside, the gaming booth automatically seals the door and activates the game. Juan has mastered playing the games in his own "secret strategy" and has not lost one yet. As a "pro-master", he bets all he has in his hands every time. This afternoon is no exception in his "planned and systematic luck". He is winning and wrecking havoc on the

game machines. The wins at the end of each game is automatically shoved out at the receiving tray exact to the last change. To get out of the booth, he has to insert another token, bought inside, which automatically opens for exit. As designed, these series of machines will only "accommodate" one game at a time with the sequence, *entrance, play, and exit*. Juan Tamad goes into a series of these gaming galleries and humbling each of his "foes", once again exemplifying the triumph of man over the machines.

There are numerous questions which can be evolved from this situation. I will leave it to the reader to formulate some of these questions for his own use and pleasures. Here are some to begin with:

1. What was the exact amount in Juan's pocket that afternoon, if he went into three of the series and came out at the end of the third booth with no amount left? Each token cost P10.00 for entrance and exit.

2. Same as (1), if each token cost P8.00 for entrance and exit.

3. What is a general formula for the amount at each booth's exit in Juan's "schema" for any k number of booths when the cost of each token is p and the original amount in his pocket is A ?

4. What is the original amount A if winning is minimal at each booth?

5. What is the rate of change per hour of the win at each k if the original amount A and the cost of each token p are constants and Juan is vanquishing five gallerias per hour?

In another episode, Juan Tamad goes *"ala - Robin Hood"*. He is "robbing and conquering" the gaming galleries at the video arcade. He is winning and raking in prize money in the galleries through his "mastery and superiority" over gaming machines. With his "prized loots", he is sharing them with the poor and the run-down multitudes, through the current craze of the time, the "5-6 *Scheme*". Juan Tamad is a believer in not giving outright "help" which tends to make people indolent, but in assistance-in-aid with corresponding responsibility and obligation. He is hailed by the common folks as a present-day hero and a stout-hearted benefactor. Where else can you go and get "help" at an instant with "no hustle and no question ask", but to our folk hero, Juan Tamad.

The barangay of *Sariling-Unlad*, once unknown and undoing, is now flourishing into a booming entrepreneurial place. With the little capital anyone gets from the *Juan Tamad's Enterprises*, every kind of little *tienda and sari-sari* stores have sprung in every little nook and corner of the once sleepy town.

Will *Sariling-Unlad* "boom or bust"? Is Juan Tamad a real benefactor or a scrupulous oppressor?

QUESTIONS

1. Show the simple arithmetic in determining the amount one has to pay a year later for a "loan" of P5.00 which is "agreed" on a "5-6 monthly scheme"?

2. The little *tienda and sari-sari* stores are financed on a 5-6 daily scheme". How much one has to pay a month later for a "loan" of P100?

3. The "5-6 Scheme" follows the principle of "natural growth" principle which states that for any amount $A(t)$, at time t, its rate of growth. $A'(t)$ is directly proportional to the amount at the time t. That is, we have a differential equation

$$A'(t) = \frac{dA}{dt} = k\,A(t), \text{ for some proportionality constant } k.$$

Solve for an explicit formula for $A(t)$ for a "5-6 daily scheme".

4. What is the rate of interest per annum for a "5-6 monthly scheme"? What about for a "5-6 daily scheme"?

5. In this episode, is Juan Tamad a "hero" or a "villain"?