Innovative Lessons for Integrating History of Mathematics in Secondary Algebra and Trigonometry

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Abstract

The use of history of mathematics in mathematics teaching is widely recognized. However, to date, secondary mathematics teachers do not find enough readily available teaching materials for this purpose. Supplemental teaching materials for integrating history of mathematics in secondary algebra and trigonometry were designed to stimulate and motivate students to learn high school mathematics and remedy the scarcity of such learning materials. These also served as models for mathematics teachers in designing curriculum materials from history of mathematics for classroom use.

Non-scheduled standardized interviews were conducted to 18 randomly selected high school students on their perception of the teaching and learning experience. Responses to the student interviews revealed that most of the students preferred mathematics with history. "History gives more meaning to mathematics concepts", "history motivates students to study more", "including history makes math easier to understand" and "no history makes math boring" were the reasons that they offered. The students judged the supplemental materials as learning materials that could increase interest in mathematics.

Keywords: Curriculum Innovations, History Integration, Trigonometry

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Introduction

Mathematics is a necessary human activity, one that responds to needs dictated by human existence itself. Mathematics is evident in all societies and cultures. Constantly evolving in response to societal conditions. (Swetz, 1984). The needs it responds to are both material and intellectual. As the needs change, so does the nature of mathematics.

Among most branches of science, the process of progress is one of both correction and extension. Aristotle, one of the greatest minds ever to contemplate physical laws, was quite wrong in his views on falling bodies and had to be corrected by Galileo in the 1590's. Even Newton, the greatest of all scientists, was wrong in his view of the nature of light, of the achromaticity of lenses, and missed the existence of spectral lines. His masterpiece, the laws of motion and the theory of universal gravitation, had to be modified by Einstein in 1916.

Mathematics history differs in existence from all other histories. What makes mathematics unique? It is only in mathematics where there are no significant corrections – only extensions. Once the Greeks had developed the deductive method, they were correct in what they did, correct for all time. Euclid's work was incomplete and has been extended enormously, but it has not had to be corrected. Every one of his theorems is valid to this day. Ptolemy may have developed an erroneous picture of the planetary system but the system of trigonometry he worked out to help him with his calculations remains correct. Each great mathematician adds to what came previously, but nothing needs to be uprooted (Boyer, 1991).

Teachers of high school geometry are confronted daily with the problem of making the subject as interesting as possible for their students. Teachers find the challenge of producing logical-deductive arguments enough to keep the students interested and motivated. Sharing with students some of the history of the development of geometry, especially introducing them to some of the great mathematicians, will get them to appreciate the fact that mathematics is the product of human minds, created in response to human needs. (Lightner, 1991)

History can explain a great deal of the "whys" in mathematics. (Bidwell, 1993) According to Jones (1969) there are three categories of whys in the teaching of mathematics. These are the chronological, the logical, and the pedagogical. *Chronological whys* include such items as why there are sixty minutes in a degree or an hour. This type looks for the source of such words as "zero" and "minute" as well as "second". These and many other historical facts, large and small, not only give answer to the whys of a particular question but also, in the hands of a skillful and informed teacher, trigger discussions about the necessity and arbitrariness of definitions and of undefined terms. It can also pave the way for discussions about the psychological bases of mathematical systems and about the development of extended definitions, such as those of "number" which change with the development of new systems, being generalized and rephrased to include the new ideas as well as the old.

Logical whys, as explained by Jones (1969), include an understanding of the nature of an axiomatic system as well as the logical reasoning and proofs that clothe the axiomatic skeleton with theorems. It is important that students grow to understand this structure. It is also consoling, on the part of the students, to find that Descartes called negative numbers "false" and avoided their use; that Gauss had a "horror of the infinite". Not only do these stories offer the consolation that great men once had difficulties with what today are fairly-well-clarified concepts. They also show how mathematics grows and develops through generalizations and abstractions. They indicate that the mathematics of a few years from now will no doubt be different from the current mathematics while still including today's ideas, perhaps in altered form.

Pedagogical whys, as pointed out by Jones (1969), are the processes and devices that are not dictated by well-established arbitrary definitions; they do not have a logical uniqueness. They include such devices as "working from the inside out" in removing nested parentheses. This is not logically necessary, but experience and a little thought suggest that they are less likely to lead to errors. Historical ideas may help in both the selection and explication of such pedagogically motivated processes.

The history of mathematics will not function as a teaching tool unless the users (1) see significant purposes to be achieved by its introduction, and (2) plan thoughtfully for its use to achieve these purposes (Jones, 1969).

Solomon (1989) provides some answers to the question "Why do we want to teach history?" The reasons she puts forward are for illustrating the nature of scientific knowledge; to show the human face of science; and for understanding the social relations of science. On the question "How shall we teach it?" Solomon provides four different methods: using contemporary sources, using drama, small group discussion work and imaginative writing and artwork.

Savellano (1996) points out that: The history of science offers a rich source of materials for learning values, for giving "humanness" to science, and for understanding mankind. Through the study of the history of science, students acquire a deeper understanding of the interaction between science and society. Science teachers can consider various strategies in teaching the history of science to underscore that science, if taught properly, will stimulate students' interest in, and develop positive attitudes towards science. Some suggested strategies to make the history of science/mathematics more effective are: use of vignettes, dramatization, small group discussions, imaginative writing and artwork. However, some guidelines are put forward and they are: remember to use - historically accurate materials; fiction to enhance the element of human interest; materials that are short and entertaining; supplementary or motivational activities; materials that stimulate and inspire discussions; and materials that emphasize a scientific attitude (pp.22-27).

According to Guerlac (1977, cited in Jenkins 1989), the 'central business of the history of science is to reconstruct the story of acquisition of knowledge of . . . science and, above all, to study science as a human activity, how it arose, developed, expanded, how it has influenced or been influenced by man's material, intellectual, even spiritual aspirations. It is a description of 'the business' which is attractive, and one, which seems to imply a major role for the history of science, not only in contributing to, and shaping the objectives of, a humane scientific education, but also in suggesting a new dimension for the teaching of history in secondary schools.

Russell (1981) illustrated how history of science relates to influencing attitudes toward science by scrutinizing the data collected by Harvard Project Physics. The developers of the Project Physics Course deliberately set out to influence students' attitudes toward a science subject and, as part of their strategy, incorporated directly into the course substantial material from the history of science. Welch and Walberg (1972, as cited by Russell 1981), summarized the significant course effects as follows:

Project Physics students rated the concept Physics as more Historical, Philosophical, Social and Humanitarian and less Mathematical and Applied than did students in other courses. Project Physics has interesting historical approach, math background unnecessary and books enjoyable to read and it has diversified learning environment while other physics classes are rated 'most difficult course in school'. In addition, no significant differences were obtained on the cognitive measures in the course comparisons (p.55).

Walker and Schaffarzick (1974, cited in Russell 1981) reviewed 23 studies and reached the plausible conclusion that teachers can influence student learning in significant ways by the content they include and the emphasis they give to the content of a course. Walker and Schaffarzick generalized about how much history of science is required to influence attitudes thus: "If we wish to use the history of science to influence students' attitudes toward science, we must include significant amounts of historical material in our course content" (Russell, 1981, p.55). Furthermore, they added as a second conclusion: "If we wish to use the history of science to influence students' understanding of science, we must include a significant amount of historical material and treat that material in ways which illuminate particular characteristics of science" (Russell, 1981, p.56).

Instructors who embrace the constructivist approach to teaching and learning will provide meaningful supporting concepts for students to enhance their understanding of a new concept under study. It is for this reason that the constructivist view of learning favors storytelling since stories provide many connections among concepts, making them more meaningful to the learner. The learner integrates the new information into existing schemata in a meaningful and interesting way. According to Wandersee (1990), one means of introducing the nature of science into science courses in a manner that will be meaningful and thought provoking is through the use of historical vignettes.

Vignettes about the history of science provide a theoretically grounded method by which the history of science can be explored. Vignettes are little stories that tell an interesting, attention-grabbing slice of bigger stories. They are fictional stories based on historical accounts of science whose function is to make science interesting, while providing important information to the students about the history of scientific developments. Vignettes will help the students make connections between the past and the present, and show them how the concepts that they are learning as "facts" were discovered and developed. Vignettes stimulate the students' interest in science (Roach, 1992).

An interactive historical vignette (IHV), on the other hand, is a technique for teaching the nature of science and mathematics. It combines a story telling method with a discussion method. These interactive stories take only about 10-15 minutes of class time; they provide a link between primal science and contemporary science. They should be used to emphasize the process of acquiring knowledge (Roach and Wandersee, 1995).

Quotations, according to Fleron (1998), may be used to stimulate journal writing, to encourage extracurricular student investigations, to introduce new topics, to give evidence for the relevance of topics, to clarify thinking, to inspire and stimulate, to increase vocabulary, to summarize key concepts, to give credence and authority to our words, and to promote thought and discussion. Quotations can also be used to amplify historical, humanistic, aesthetic, intellectual, artistic, philosophical and epistemological aspects of mathematics.

Using costumes and drama can add a special appeal to looking at the history of mathematics. The students not only learn the human traits of the mathematicians but they can also see a living person acting out these traits. The mathematician's work can be highlighted in ways that are understandable and memorable. Dramatization can be as short as a given appearance that lasts only a few minutes, or it can be a fully scripted play (Shirley, 2000).

Mortimer Adler (1982, cited in Roach 1992) recommends the use of these to stimulate and motivate students and open doors to the world of learning. It is on this context that this study was conceptualized.

Objectives of the Study

This research aimed to develop and validate supplemental teaching materials (in history of mathematics) for fourth year high school mathematics, which integrate history of mathematics with the lessons in algebra and trigonometry toward enhancing the affective learnings of mathematics students. Moreover, it aimed to determine the perception of high school students about the integration of history of mathematics in the teaching of algebra and trigonometry by answering the following interview questions: How do you prefer your mathematics to be taught – the mathematics lessons with history or the mathematics lessons without history? What can you say about the reading (supplemental) materials given to you? Which do you like best or least among the lessons about history of mathematics? In your opinion, what are the characteristics of a mathematician?

Significance of the Study

This study focused on researcher-developed supplemental teaching materials that can be used by mathematics teachers in integrating history of mathematics for classroom use. Such instructional materials may prove to be the answer to the question: What is the best way to stimulate and motivate students to learn high school mathematics?

In like manner, the lessons will make the students aware of the power and limitations of mathematics. Using history in teaching mathematics provides a personal and cultural context for mathematics, which helps students sense the larger meaning and scope of their studies. When they learn how persons discovered and developed mathematics, they begin to understand that posing and solving problems is a distinctly human activity. Moreover, by viewing mathematics from a historical perspective, students learn that the process of problem solving is as important as the solution. As a result, students' perspective of mathematics can shift to a more positive one. It is certain that mathematics will play an even greater role in their future and there will be many occasions when they will need to make judgments and proper choices.

Scope and Limitation of the Study

The researcher constructed and validated the supplemental materials integrating the history of mathematics in line with the topics in Mathematics IV (Algebra and Trigonometry).

The supplemental materials include biographical information of mathematicians, anecdotes of known inventions of mathematicians, quotations from mathematicians, and vignettes on the development / discovery of important mathematics concepts.

Theoretical Framework

Science education literature has established the need for inclusion of the history and philosophy of science (nature of science) in high school science courses (e.g., Duschl, 1990; Matthews, 1990; Wandersee, 1990). This is also true with mathematics education as evidenced in the works of Smith (1951); Hofman (1959); Swetz (1984, 1989); Boyer (1991); Jones (1991); Lightner (1991); and Bidwell (1993).

The National Council of Teachers of Mathematics (NCTM) Standards' first "New Goal for Students: Learning to Value Mathematics" (1989) states, in part:

Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves. It is the intent of this goal-learning to value mathematics – to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed and the impact that interaction has on our culture and our lives (p.5-6).

It is evident from the above statement that there is also a need for the integration of history of mathematics in the mathematics courses so that students will value the discipline.

The constructivist theory defines learning as an active, constructive process of the learner's current knowledge and the new knowledge presented. It assumes that the learner comes to the classroom with strongly held beliefs and views about everything in this world. These prior beliefs or conceptions are found not only difficult to change but they also impede learning. Thus, learning is also viewed as a process of conceptual change.

Studies showed that strategies/approaches that are anchored on the constructivist theory of learning have potentials to improve students' conceptual understanding. Teachers who embrace the constructivist approach to teaching and learning provide meaningful supporting concepts for students to enhance their understanding of a new concept under study. It is for this reason that the constructivist view of learning favors storytelling or vignettes/fictional stories based on historical accounts of mathematics (Roach, 1992); quotations used to amplify historical aspects of mathematics (Fleron, 1998); anecdotes of known inventions of mathematicians (Bidwell, 1993) and the history of the development of Geometry (Lightner, 1991) since these materials provide many connections among concepts, thus making them more meaningful to the learner. Moreover, the learner integrates the new information into existing schemata in a meaningful and interesting way (Wandersee, 1990) and thus enhancing his understanding of the concept.

On the other hand, the information processing theory explains the limited capacity of the Short Term Memory (STM) where processing of information and "learning" takes place. This limited capacity of the STM is believed to inhibit learning. Thus, this theory suggests that if the memory load of the learner can be reduced, then the learner can attend to more important aspects of the learning process. The integration of history of mathematics reduces the memory load by presenting other aspects. This is best accomplished by "brief, stimulating, historical lessons" (Wandersee, 1990) interspersed with concepts throughout an existing mathematics course.

Construction of Supplemental Materials

Supplemental Materials for Algebra and Trigonometry are instructional materials for Algebra and Trigonometry incorporating History of Mathematics that included biographical information, and quotations from mathematicians, anecdotes on known inventions of mathematicians, vignettes based on the lives of great mathematicians and the development/discovery of important mathematical concepts. These materials were collected, constructed and validated by the researcher.

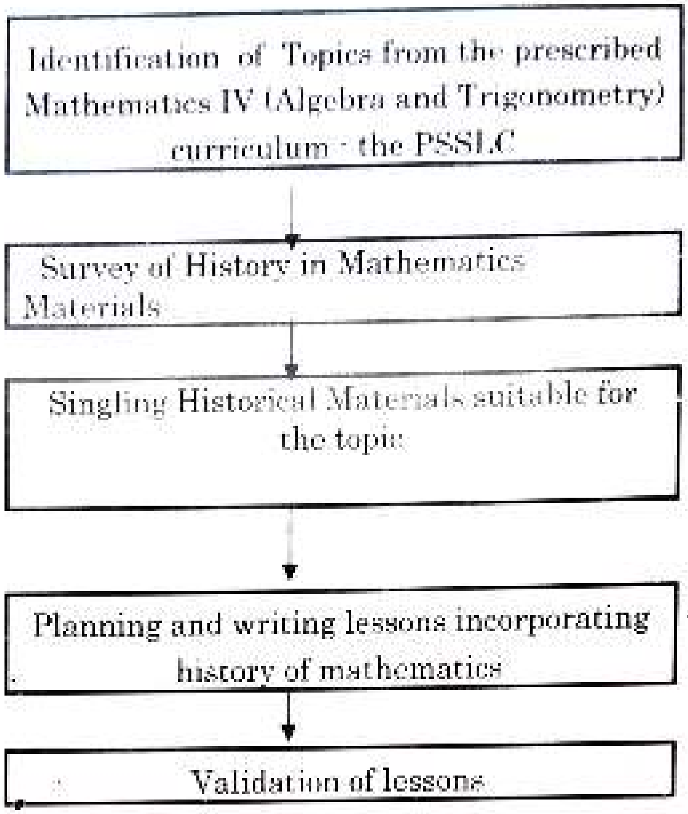

Figure 1. Process in Constructing the Supplemental Materials

In the making of the supplemental materials, the researcher had followed the process as diagrammed in Figure 1. The first step was to look into the Mathematics IV (Algebra and Trigonometry) curriculum for fourth year students as prescribed by the Department of Education. The Philippine Secondary School Learning Competencies (PSSLC) for Secondary Education Development Program was referred to in identifying the topics. The topics considered in this study were enumerated in the limitation of this study. After the identification of topics, survey of existing historical materials was done. In the review of historical materials, the following were taken into consideration: the development of mathematical concepts, vignettes, biographical data of mathematicians, anecdotes, quotations, problems of history of mathematics, and activities. The sources for historical materials came from the internet, history of mathematics books and mathematical journals. In singling out materials suited for a topic, the researcher considered the available materials gathered and the materials that would best fit the topic. For example, in the topic "Length of an arc", the researcher had chosen the historical account of 'why there are 360 degrees in one revolution', 'how did radian measure come about' and 'the origin of radian measure' because degrees and radians are measures for the length of an arc. The planning of the lessons came next. Historical materials could be placed in the first part of the lesson for motivational purposes, in the middle as part of the lesson and at the end for assessment. After the writing of the lessons with the integration of history, these lessons were shown to experts in the field of mathematics education and high school mathematics teachers for comments and suggestions. After revisions, these were pilot tested.

The researcher also constructed lesson plans for Algebra and Trigonometry showing the integration of history of mathematics based on the Philippine Secondary Schools Learning Competencies (PSSLC) for fourth year mathematics. The lesson plans included topics on trigonometric and exponential functions. There were 15 lesson plans made by the researcher intended for the third grading period. These lesson plans were also shown to experts in mathematics education and mathematics teachers for comments and suggestions. These lesson plans were pilot tested to fourth year students and revised based on the given comments. A sample lesson plan is shown in Appendix A. The list of lesson plans constructed by the researcher is also shown in Appendix B.

Data Collection

After the presentation of the lessons, the researcher did some informal interviews with students. She had interviewed 18 randomly selected students (9 belonging to the high ability group and the other 9 came from the low ability group). The non-scheduled standardized interview technique (Goetz and Le Compte, 1991) was used, where almost the same questions were used for all the interviewees. The order in which the questions were posed changed depending on how the respondent reacted. In some instances, some questions from the guide questions were skipped. In other instances, the interview was cut short when the respondent showed disinterest as indicated by consecutive response of "*nakalimot ko*" (I forgot) or "*wala ko kahibalo*" (I don't know). To increase the accuracy of data recording, the interview was audio taped and transcribed. Each interview session lasted for about 30 minutes.

In the presentation of the qualitative data, codes were used for the students. The high ability group was coded H while L was the code for the low ability group. M is the code for male subjects while F for females. Thus, the code LM01 means "Low ability, Male, student 1" or HF13 means "High ability, Female, student 13".

Students' Perceptions about the Integration of History of Mathematics in Algebra and Trigonometry

To assess students' perception on "how well they received the new way of teaching" and "how well they received the supplemental materials on integrating history of mathematics", several questions were asked in the interview:

1. History of mathematics was included in the lessons in Algebra and Trigonometry during the third grading while during the first and second grading, there was no history included. How do you prefer your mathematics to be taught – the mathematics lessons with history or the mathematics lessons without history? Why?
2. What can you say about the reading (supplemental) materials given to you?
3. Which do you like best or least among the lessons about history of mathematics?
4. In your opinion, what are the characteristics of a mathematician?

Eighteen (18) students participated in the interview. Nine students (5 males and 4 females) came from the high ability group while another nine students (4 males and 5 females) represented the low ability group.

Most of the students (16 out of the 18) gave an affirmative response to question 1. They preferred the teaching of Algebra and Trigonometry with history of mathematics. Their reasons were varied as the following excerpts show:

1. History would give background to a concept thus giving it more meaning.

> *"I prefer math with history because history will give math concept some background so that we can understand it easily and will give more meaning to the concept just like 'sine'.* (HM02)
>
> *"Kay mahibaw-an nato ang beginning o history sa math, kung kanus-a nagsugod ang math concept ug kinsa ang nag-discover ini.* [Because we would know the beginning or history of mathematics, when a math concept started and who discovered it]."(HF01)

2. Not knowing the history would make one ignorant.

> *"I am in favor of math with history. It is good because even if you know the formula or you know how to compute but don't know where those come from or who formulated it, we are still somewhat ignorant. Having math with history will give students more knowledge. By more knowledge, I mean kanang kahibalo na ka sa math unya naa pa gyud bonus kun aha kana gikan* [I mean that you already know mathematics and you also got a bonus by knowing where it came from]."(HM10)

3. Knowing history of mathematics is like knowing your family tree.

> *"I prefer the teaching of math with history because history is important to us. It gives us more idea of our origin. Parcha bitaw sa family, ma'am, kinahanglan nga mahibaw-an mo ang imong kagikan o history para naa kay idea sa imong family tree.* [Just like a family, ma'am, it is important to know your origin or history so that you have an idea of your family tree]." (HM08)

4. Including history of mathematics motivates studying of mathematics more and getting good grades

"I prefer math with history because my grades are good and high. I like the lives of the mathematicians, their characteristics and their contributions like Pythagorean Theorem, Cartesian plane and the circle. It motivated me to study more." (HF10)

"Ang akong gusto kanang na'ay history kay daghan ug discussion unya makabalo ka sa life sa mga mathematicians. Giganahan ko pagto-un busa dili na ko mazero kay naa man gyud koy ma-answer. [I prefer mathematics with history because of the discussion and you would know the lives of the mathematicians. I was motivated to study that is why I no longer got zero because you have something to answer.]." (LF06)

5. Including history makes mathematics easier to understand.

"Ganahan ko ug math karon nga na'ay history kay sayon ra ug maka-answer pa ko kang ma'am. [I like mathematics now with history because it is easier and I could answer our teacher.]" (LF01)

"Gusto ko sa math nga na'ay history kay lahi ra man kaysa walay history. Sayon ra karon kaysa una. [I prefer mathematics with history because it's different from mathematics without history. It's easier now than before.]" (LM01)

6. Knowing history means learning about the lives of mathematicians, their stories and quotations.

"I prefer math with history because the concepts are still the same but lives of mathematicians were included together with their stories and quotations." (HM17)

> *"I would prefer math with history because we need math to learn more and we know how math developed through the years. I thought that mathematicians are perfect dili man diay* [it's not], *they are not really perfect."* (HF13)

7. Not including history of mathematics is boring

> *"Gusto ko na'ay history, kay kung walay history sumo. Diri-diritso ang leksyon namo. Pero na'ay history, daghan ko ug mahibaw-an bahin sa mga mathematicians, kinsa'y nag-discover sa formula nga among gigamit ug kinabahi nila.* [I prefer (mathematics) with history because it is boring if we don't have history. If history is included, I will know about the mathematicians, their lives and their discovery.]" (LF07)

However, not all students were affirmative in the integration of history of mathematics in Algebra and Trigonometry, as gleaned from the response of a student from the high ability level:

> *"I would rather favor more the teaching of math without history especially that the achievement test is near. Hinay man gyud ang na'ay history* [the pace of the lesson is slow when there is history] *and I am afraid that we would not finish the math topics that will be included in the examination"* (HF09).

The responses to the students' interview on Question 2 – "What can you say about the reading (supplemental) materials given to you?" revealed that most of the high ability students in the experimental group judged the supplemental materials by whether it could increase interest in or motivate more learning of mathematics. They said:

1. The supplemental materials are interesting and motivate one to read.

"I like the anecdotes. I was motivated to read them because of the pictures. The anecdotes are very funny and informative."(HM02)

"Naa tanan ang information [All information are there], *interesting, it motivates you to read and the anecdotes are short but kataw-anan* [funny]. "(HF10)

2. The supplemental materials are easy to read.

"The materials are easy to read and give us information about mathematics and how they invented something. Ang mga anecdotes ma'am kay tinuod baya [The anecdotes are really true, ma'am.]. "(HM08)

3. The supplemental materials are enjoyable.

"The materials are perfect because all is there. It is compact, the summary and the generalization were good. The anecdotes were informative, funny and easy to understand but with a lesson on math for you to think about. I enjoyed it very much."(HM07)

"Mas na-enjoy gyud ako ma'am [I enjoyed it, ma'am.] *If you had noticed I always raise my hands and I always want to answer our teacher because I had understood so much Unya na-enterested pud ko sa mga mathematicians nga apil sa mga lessons.* [I was also interested with the mathematicians included in the lessons.] "(HM10)

4. The supplemental materials are exciting.

   "I read the materials you had provided us at home. I even read it to my sisters especially the anecdotes. The materials are very exciting. They can add more knowledge."(HF13)

5. The supplemental materials are understandable.

   "The reading materials were understandable, interesting and easy to read. I even spent time reading them at home."(HM17)

6. The supplemental materials are very good and in detail.

   "The reading materials were very good. They explain well how mathematicians discover their contributions. Na'ay proof nga gipakita, pananglitan, ang Pythagorean theorem kay gagamit sa Pythagorean puzzle aron ma-prove ang theorem.[There was a proof of the Pythagorean theorem shown using the Pythagorean puzzle]. *The reading materials were in detail and interesting. I like the quotations because it caught my attention. I even find time reading the materials at home."*(HF01)

The experimental low ability group of respondents, on the other hand, did not welcome the supplemental materials. Most of them read the materials inside the class only. Among their reasons were:

1. So much work at home and no time to read.

   "Sa school na lang ko naghasa kay namaligya man gud mi ma'am sa Kiwalan ug wala'y magbantay sa tindahan. [I read in school because we had a store in Kiwalan, ma'am, and I watched our store]"(LM02)

   "Akong gibasa ang mga materials ug 10 minutes lang kay daghan man ko ug trabaho sa balay sa akong

> *auntie. Diha man gud ko gapuyo sa ila.* [I read the materials for 10 minutes because I have so much work at my auntie's house.] "(LF01)
>
> "*Kay Descartes lang ang akong nabasa kay wala man gud ko'y oras sa balay tungod sa trabaho. Magbasa na lang ko dinhi sa classroom pero gamay na lang ang oras kay magklase naman.* [I only read Descartes because I have no time at home because of work. I only read inside the classroom for a few minutes because it's already time for class.] "(LF17)

2. The supplemental materials are wordy.

> "*Wala nako mabasa sa balay kay dili ko ganahan magbasa ug daghan letra. Mapugos lang ko ug basa kun tawagon ko ni ma'am sa class.* [I was not able to read it at home because I don't like to read with so much script. I'm only forced to read when ma'am calls me to read in the class.]" (LM08)

One student in the low ability level, LM06, was not able to read all because he lost the reading materials.

The low ability group of students was not happy about the extra reading materials. It can be deduced that they don't like reading. They have many reasons for not reading the materials at home. A positive comment coming from one student of this group was:

> "*Wala nako mabasa ang mga basahon pero gitan-aw nako ang tanang mga pictures sa mathematicians kay nindot man. Medyo pareho ug nawong si Pythagoras ug Thales.*" [I was not able to read the reading materials but I did look at the pictures of mathematicians because they are attractive. Pythagoras and Thales look alike]" (LM01).

This comment showed that some students in the low ability group enjoyed looking at the pictures included in the supplemental materials but not the text print.

The respondents from both the high and low ability levels were asked during the interview which lessons in mathematics they liked best or liked least (question 3). Their answers were varied. Most of the high ability group liked the history part. A student expressed his fondness for the anecdote about the donkey:

> *"I liked the anecdote about the donkey because the donkey thought that it could always fall into the river so that its burden, a sack of salt, will be light. It did not know that Pythagoras would change the content of the sack into a sponge and when it falls again into the river, its burden became heavy. I like the lesson of the anecdote."* (HM07)

The students were interested in the lives of mathematicians

> *"I liked Pascal because he invented the first calculator that leads to the scientific calculator now and for the students it's really needed."* (**HF**10)
>
> *"I liked the anecdote about Newton's love for the cat and its kitten at the farm. It shows that he is only human."* (HF09)
>
> *"Ganahan ko ni Pythagoras, bale nagtudlo siya sa mgs estudyante nga iyang gibayran. He liked to share his knowledge with others bisag siya ang nagbayad. Unya ma-relate ni nako sa akong mga parents nga kung taas ang akong grado taga-an nila ko ug incentive.* [I liked Pythagoras because he paid the students so that he could teach them. He liked to share his knowledge even though he was paying them. I could relate him to my parents. My parents give me incentives whenever I get good grades.]" (HM10)
>
> *"I liked Pythagoras and Bascara. Pythagoras gave us the Pythagorean Theorem and Bascara made a proof of the Pythagorean Theorem using squares and triangles just like the activity we did in the classroom."* (HF09)

One student expressed her aspiration to be like Pythagoras:

> *"I would like to be Pythagoras because he is a good teacher and he likes to teach even if he will pay his students. I like to be a teacher but I don't have money to pay my students. I would like to mention to my students someday what Pythagoras did so that they will appreciate math just like what I did."* (HF13)

Most of the students, however, expressed their dislike for the lesson on Surya Siddhanta (an Indian book which contains a table of numbers recorded in columns S_n and V_n ; if plotted, the sequence S_n looks like a quarter of a sine wave (Kennedy, 1969, p.347). Here are their views about the lesson:

> *"I got confused with the formula of Surya Siddhanta and I only solved the 5th term of S_n even though I have a calculator."* (HM10).
>
> *"I don't like the lesson on Surya Siddhanta because of the long computations to get the next term of the S_n."* (HF01)

Most students interviewed discerned that mathematicians are observant, open-minded, curious, imaginative, persistent, intelligent, and accepts mistakes. All these characteristics are in answer to question 4 – "What are the characteristics of a mathematician?"

> *"Mathematicians are keen observers, bright because they are able to think, apply and think of possible ways that makes a thing work. They are people like us but with patience to let something happen. They are persistent and nature lovers just like Newton who first made his experiment by going against the wind to measure wind force."* (HM07)
>
> *"Mathematicians are problem solvers. They create something, just like Descartes with his x and y of the Cartesian plane."* (LF07)

Implications in Mathematics Teaching and Learning

Students' responses to the introduction of history of mathematics were dichotomized. Some felt it increased the interest to learning and some said that it made learning even boring. Students' interviews reflected that the sole concern of the students is interest. In other words, students would judge a new way of teaching attractive or not by whether it makes learning stimulating and interesting. Along this line, those students who are scared of too many words would find historical stories boring rather than the other way around. One way to lessen this "language anxiety", according to Chi–Kai Lit et. al (2000), is to think of introducing the historical stories orally (by narration) instead. Written texts can be distributed as optional reading for students to read after class at leisure. This is in line with previous research that person–environment fit (congruence between actual and preferred classroom environment) is the core of making classroom environments conducive to learning (Wong and Watkins, 1996).

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and the *Basic Education Curriculum* (DepEd 2002) emphasized the need to address linguistic fluency or communication skills, which include reading, writing, listening and speaking, to enhance mathematical understanding and problem solving ability. The students' passivity in mathematics classes is one basic problem that confronts mathematics teachers. Some teachers usually get disappointed due to the fact that students would not bother asking questions even if they did not understand the lesson. The question that some mathematics teachers ask, according to Role (1995), is "Why could we not get our students to talk and ask questions in our classes?"

Mathematics deals with abstract concepts and many times, students do not even know what questions to ask about the concepts. To encourage them to talk, the teacher should provide the students some talking points. From what the researcher has gathered from her observations, historical resources related to mathematics are close to the hearts of the students and this is a fertile ground for effective student – teacher and student – student interaction. When students get used to talking and expressing their ideas and opinions in historical discussions, this will be carried over in the actual discussions of mathematics lessons; thus, students will be free to express how they feel about the subject matter itself. This might stimulate their thinking that would lead them to ask questions about mathematics.

One very important word of caution to teachers leading out in historical discussions: *There is a tendency for students and teachers to be so carried away by the interesting discussion of historical resources that they might forget their immediate concern, mathematics learning.* The teacher using the integration of history of mathematics in teaching mathematics should be cautious about this so that time will not be wasted. She should learn to limit the discussion within the bounds of the lesson.

Both observation and research show that another reason why Filipino students do not talk in mathematics classes is because of limited English-language proficiency. Students with language background different from English need special help in adapting to a mathematics class conducted in English (Role, 1995). "Students who do not possess the level of English proficiency needed for meeting academic requirements lack some or all of the wide range of skills. Some students lack the ability to listen, speak, read and write in English. Other students have speaking and listening skills but are below high school level in reading and writing" (Cuevas, 1990). The students who participated in this study have these typical characteristics of second-language learners.

What can teachers do to provide opportunities to enhance communication skills in mathematics? What can be done to help these students participate meaningfully in class activities? How can students be assisted in developing the language skills they need to deal with the tasks and materials given in class? Role (1995) suggested some instructional strategies for dealing with these challenges. Two of these strategies which were used in this study were found effective: (1) *Offer opportunities for students to talk about mathematics using their native language as well as English and (2) maintain a classroom climate in which students with limited English proficiency are encouraged to participate.*

The teacher who participated in this study offered opportunities for students to talk about mathematics and historical issues related to mathematics using their native language as well as English. Research findings suggest that concepts and skills can be reinforced when language-minority students are given the chance to discuss them in their native language (Hakuta, 1986).

Recommendations

The present study reports the designs of supplemental materials for integrating history into algebra and trigonometry for high school students as well as the process of its experimentation. The process of its development should shed light on how a supplemental material in history of mathematics could be gradually constructed based on situational appraisal of learning difficulties. It also serves as an example of latter designs. Students' feedbacks will provide rich information for improvements. Eventually, the teacher would not only depend on ready-made teaching materials but gradually develop the capability of designing curriculum materials that serves the purpose of learning best. "The ideal teacher knows well his/her particular discipline, but is ready to transcend it." (BEC, 2002 p.9). In this case, the teacher should become a thinker, an evaluator as well as a curriculum designer. (Wong and Su, 1995; cited by Chi-kai Lit, 2000, p. 47). The development of curriculum concepts through history should be of great help in designing the teaching of mathematics.

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