Effects of Integrating History of Mathematics on High School Students' Cognitive and Affective Learnings

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Abstract

The study looked into the relative effects of integrating History of Mathematics in the teaching of Algebra and Trigonometry on the cognitive and affective learnings of fourth year high school students. Furthermore, it tried to find out the relative contribution of ability level and gender on their mathematics achievement, attitude towards mathematics, perception of the usefulness of mathematics, confidence in learning mathematics, mathematics anxiety and beliefs about mathematics.

Some 118 fourth year high school students enrolled in the Secondary Education Development Program of the Iligan City East High School were the subjects of this study. These students composed the four sections of Math IV (Algebra and Trigonometry) which were taught by one teacher. Two groups composed of one high ability section and one low ability section, randomly chosen as the experimental group, were taught Algebra and Trigonometry with the integration of History of Mathematics while the other two groups were taught Algebra and Trigonometry the traditional way. Fifteen lessons for each group were prepared by the researcher.

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At the beginning of the second semester, the learners were given the Otis Lennon Ability Test the scores of which matched the learners between the experimental and control groups. At the same time, Mathematics Achievement test, Attitude towards Mathematics Inventory, Perception of the Usefulness of Mathematics Scale, Confidence in learning Mathematics Inventory, Mathematics Anxiety Scale and Inventory on Beliefs about Mathematics were also administered. These instruments were also used as posttests.

The quantitative findings of the study revealed no significant difference in the achievement of learners taught Algebra and Trigonometry with history of mathematics and those learners who were not taught with history of mathematics across ability and gender. However, high ability learners exposed to the integration of history of mathematics perceived mathematics more useful than those taught Algebra and Trigonometry without history. The learners' attitude towards mathematics, confidence in learning mathematics, mathematics anxiety and beliefs towards mathematics did not differ among different ability levels and gender before and after the experiment. Furthermore, the integration of history to the teaching of mathematics was more beneficial to high ability students.

The achievement and affective learnings were interrelated such that: A learner who has a high achievement in mathematics has a positive attitude towards mathematics, less anxious, perceived mathematics to be useful in real-life situation, has improved beliefs about mathematics and is confident to learn mathematics.

The qualitative results yielded improved learners' attitude and views about mathematics, positive perception of the usefulness and beliefs about mathematics, made students more confident to learn mathematics and lessened anxiety because of the integration of mathematics in the teaching of Algebra and Trigonometry.

Keywords: History Integration, Cognitive Learning, Affective Learning, Beliefs

Introduction

The current teaching of mathematics is one-dimensional. As Swetz (1984) observes, mathematics teachers frequently concentrate on the symbols, the mechanics, the answer-resulting procedures – without really teaching what mathematics is "all about", where it comes from, how it was labored on, how ideas were perceived, refined, and developed into useful theories. In brief, the social and human relevance of the subject is not brought into place.

At best, this method of teaching mathematics produces knowledgeable students who can dispassionately use mathematics but who at the same time perceive mathematics as an incomprehensible collection of rules and formulas that appear *en masse* and threateningly descend on them. They think that mathematics is "closed, dead, emotionless and all discovered. It lies completely in a book or in the mind of the teacher ready to be rolled out and absorbed" (Bidwell, 1993, p.461). Thus, students build in themselves psychological barriers to true mathematical understanding and develop anxieties about the learning and use of mathematics.

Acknowledging the problem posed by this situation in mathematics teaching, the Curriculum and Evaluation Standards for School Mathematics (1989) of the National Council of Teachers of Mathematics (NCTM) has called for considerable reform in the approach to teaching and learning mathematics. Three key items reconsidered examined across all grade levels are communicating, connecting and valuing mathematics. History allows us to study all three. Students can communicate about historical facts orally or in writing; they can connect mathematics to various cultures as well as to other intellectual developments in science, philosophy and religion; history can substantially add to students' value of mathematics learned from the past and in the present.

What can the history of mathematics do for the students? When properly used, coupled with an up-to-date knowledge of mathematics and its uses, History of Mathematics is a significant tool (Jones, 1969; Wandersee, 1990) in the hands of a teacher who teaches the "whys" (be it the chronological whys, the logical whys or the pedagogical whys); teaches for meaning and understanding; teaches so that children see and appreciate the nature, role, and fascination of mathematics; teaches so that students know that men are still creating mathematics and that they too may have the thrill of discovery and invention.

On a more practical level, diversifying the manner in which concepts are presented breathes fresh air into instruction. The teacher who uses a variety of strategies to help students learn concepts may be able to hold the attention of students at a high level. According to Park and Lamb (1992), the case for employing lessons like historical vignettes is both theoretically and practically strong.

The history of mathematics, moreover, can substantially humanize mathematics. Bidwell (1993) explores three ways of using history in the classroom. First, he considers an anecdotal display that uses pictures of mathematicians or calendars with birthrates in the classroom. Postage stamps illustrating mathematics or mathematicians as well as collections of word or symbols derivations can be displayed. These items can get students' attention and spark their interest. A second way to use history according to him is to inject anecdotal material as the course is presented; the third use is to make historically accurate developments of topics a part of the course.

On the other hand, teachers can demonstrate that the history of mathematics illuminates today's mathematics and attempt to whet their students' intellectual appetites for more. Teachers can direct them to sources appropriate for their students' current interests and powers of comprehension. Teachers can lead the students toward the horizon and invite them to follow through. Wandersee's (1990) experience indicates this is best accomplished (below the graduate level) by brief, stimulating, historical lessons interspersed throughout an existing mathematics course.

The history of mathematics appeals to human interest and attention. It shows the relevance of mathematics to life; it gives students confidence when they associate themselves with the mathematicians who were not always successful in their tasks. This way, history of mathematics motivates students to learn since learning takes place in people who are motivated (Gagne, 1988).

Learning mathematics is a cognitive endeavor. As in other cognitive fields, affective factors play an important role in it. The role of affect seems to be particularly important in the development of higher-order thinking skills especially in mathematics (Role, 1995).

It is along this line that this research is conceptualized. If affective factors play an important role in both mathematics teaching and mathematics, then it stands to reason that integrating history in the mathematics lessons can affect students' performance.

Statement of the Problem

This study aimed to determine the effects of integrating history of mathematics on fourth year high school students' cognitive and affective learnings in mathematics. More specifically, this study sought answers to the following questions: (1.) Do the achievement scores of those taught with mathematics only and those with integration of history of mathematics differ significantly (a) between students of high and low mathematics ability and (b) between male and female students? (2.) What effect does the integration of history of mathematics have on students' affective learnings, specifically in their (a) attitudes towards mathematics, (b) perceptions of the usefulness of mathematics, (c) confidence in learning mathematics, (d) anxiety in mathematics and (e) beliefs about mathematics? (3.) Does the affective learning of those taught mathematics with its history differ significantly from students not exposed to history of mathematics? (4.) In either approach, what is the relationship of mathematics achievement to the affective learnings?

Significance of the Study

This study has special relevance to the teaching and learning of mathematics since the study was conducted in real classroom situations with secondary students participating. The supplemental materials developed for this research were integrated and applied to the teaching of Algebra and Trigonometry in the high school.

Using history in teaching mathematics provides a personal and cultural context for mathematics, which helps students sense the larger meaning and scope of their studies. When they learn how persons discovered and developed mathematics, they begin to understand that posing and solving problems is a distinctly human activity. Moreover, by viewing mathematics from a historical perspective, students learn that the process of problem solving is as important as the solution. As a result students' perspective of mathematics can shift to a more positive one. It is certain that mathematics will play an even greater role in their future and there will be many occasions when they will need to make judgments and proper choices.

The results of this study could make mathematics teachers realize alternative approaches or strategies to making the teaching of mathematics interesting. Integrating history of mathematics with the usual mathematics lessons is in line with the true nature of mathematics and the real nature of mathematics learning. Teachers can adopt the strategy modeled in this study toward raising interest in, and in the understanding and appreciation of mathematics.

The result of this study might also guide mathematics supervisors and school administrators to alternative strategies in mathematics teaching.

Scope and Limitation of the Study

This is a quantitative research with qualitative support. Patton (1990, p.188) asserts, "A combination of quantitative and qualitative methods of research can achieve triangulation". The use of triangulation provides cross data validity checks. Triangulation contributes greatly to the research findings regardless of whether the qualitative or quantitative method has been employed (Borg and Gall, 1989, p.393). This study is open to the limitations of a quantitative - qualitative research. It was limited to four sections of fourth year SEDP high school students of a public high school in Iligan City. It made use of the interactive historical vignettes as strategy for integrating history of mathematics. The orientation to the strategy of the teacher handling the experimental classes lasted for two weeks prior to the actual start of the experiment. The validation of the strategy was limited to one grading period, which lasted for seven weeks starting from November 12, 2001 to January 18, 2002 for a total of 16 classroom – teaching hours. The data were confined to test results, observations, compositions of students in their journals and interviews of students.

Research Methodology

The Sample

A public high school in Iligan City of Lanao del Norte was the locale for this research. The subjects of this study are the bonafide fourth year high school students of the school enrolled in academic year 2001-2002. Four intact classes were selected from the six (6) sections in this school: two classes comprised the experimental group and the other two, represented the control group. Since the sectioning of this school was based on grade point average, the first two sections represented high - ability group of students while the last two sections represented the low - ability group. The experimental groups were sections I and III while the control groups were sections II and IV. The selection was done through casting of lots.

Comparability of the four sections in terms of ability was established using their Otis Lennon School Ability Test (OLSAT) scores administered to the students on September 17, 2001. To classify the students into experimental high and control high groups, the students of sections I and II were matched according to the OLSAT scores and gender. For example, a male student in section I with an ability score of 39 and a stanine score of 5 was matched to a male student in section II with the same ability score of 39 and a stanine score of 5. In like manner, a female student in section I with an ability score of 40 and a stanine score of 5 was paired to a female student of the same ability and stanine score in section II. The ability scores of sections I and II range from 27 to 52 with stanine scores ranging from 5 to 6, which means that these students were of average ability. On the other hand, classifying students in sections III and IV into the low ability experimental and low ability control groups was also done through pairing. A male student of section III with an ability score of 11 and a stanine score of 1 was matched to a male student in section IV with the same ability and stanine score. The same process was done with the females of sections III and IV. The ability scores ranges from 9 to 26 with stanine scores ranging from 1 to 3. These students belong to the low and below average ability. In all, there were 118 fourth year high school students included in the sample. All the four sections had more females (59.32%) than males. The groups were therefore comparable in terms of their ability at the start of the experiment. Table 1 shows the number of samples grouped according to ability, treatment and gender.

Table 1. Frequency Distribution of Samples by Treatment, Ability and Gender

GROUP	Male	Female	Total	%
Experimental	24	35	59	50.00
High	13	18	31	26.27
Low	11	17	28	23.73
Control	24	35	59	50.00
High	13	18	31	26.27
Low	11	17	28	23.73
Total	48	70	118	100.00

Research Design

This study used intact classes since all sections in the locale of the study are intact or existing sections. The research design employed was quasi-experimental. The nonrandomized control group pretest-posttest design was used. This design is appropriate since random assignment to groups is not possible (Leedy and Ormrod, 2001, p. 238)

The experimental group was taught algebra and trigonometry integrating history of mathematics while the control group followed the usual teaching of algebra and trigonometry without integrating materials on the history of mathematics. The inventories on affective variables such as attitude towards mathematics, perceptions of the usefulness of mathematics, confidence in learning mathematics, mathematics anxiety and beliefs in mathematics were administered to the students twice, as pretest at the beginning of the grading period and as posttest at the end of the grading period. The administration of these inventories was done during a one week period. For academic assessment, the scores on the Achievement test in Algebra and Trigonometry were treated as pretest and at the same time as posttest.

One teacher taught the four classes. She has taught in the public high school for 8 years. She is fully convinced of the idea of integrating history of mathematics in the teaching and learning of Algebra and Trigonometry. She was practicing integration of history of mathematics in her mathematics teaching, though not as deliberately as was done in this study. The researcher trained the teacher in the execution and use of the supplemental materials.

The Instruments

Inventories / Scales for Measuring the Affective Variables: The following instruments were used to measure the affective variables included in the study.

The Attitude towards Mathematics Inventory consisted of 28 items. Of the 28 items, the researcher constructed 13 items, conceptualized through readings and shown to the adviser for comments. The fifteen (15) other items were adapted from the Attitudinaire of Morco (1994). The items from Morco were selected on the basis of aspects not found in the researcher's inventory. After that, the instrument was pilot tested on September 10, 2001 on another class of fourth year students enrolled in the school year 2001-2002.

Fennema-Sherman Scales (1976) that include the Mathematics Anxiety Scale, Confidence in Learning Mathematics, and Perceived Usefulness of Mathematics were validated in one fourth year class for readability and use among Filipino students on September 12 - 14 of 2001. All piloted items were included in the final scales used in this study.

The researcher constructed the Beliefs in Mathematics Inventory on the basis of readings from Carter's (1997) study. Again, the same class was used to validate the Inventory. The Beliefs Inventory consisted of 14 items.

Lesson Plans for Algebra and Trigonometry showing the integration of history of mathematics. The researcher constructed these lesson plans based on the Philippine Secondary Schools Learning Competencies (PSSLC) for fourth year mathematics. The lesson plans included topics on trigonometric and exponential functions. There were 15 lesson plans made by the researcher intended for the third grading period. These lesson plans were shown to the research adviser and co-adviser for comments and suggestions. These lesson plans were pilot tested to fourth year students with the same characteristics as the subjects of the study in August 2001. These lesson plans were revised and later was shown to the teacher who would handle the experimental and control groups.

Achievement test. The researcher constructed an achievement test for Algebra and Trigonometry based on a researcher prepared table of specification. The test had two parts. The first part included items on trigonometric functions while part two included items on exponential functions. Each part consisted of 20 items. The achievement test was shown to the researcher's co-adviser and 2 fourth year mathematics teachers of the Integrated Developmental School for comments and suggestions. The test was revised after taking into consideration their comments and suggestions. The achievement test underwent validation using one class of fourth year students. Since the class time for mathematics in the school is for 40 minutes only, the researcher after consulting with the adviser decided to validate the test by parts. Part one was administered on September 5, 2001 and part two on September 6, 2001.

Student journals. A major source of data for this study was the journal writing of the students in both the experimental and control groups. These journals include the structured, descriptive and objective notes of the students. These dealt with compositions reflections, narrations and essays which the student had written about recent experiences, impressions, motives, thoughts and feelings.

Data Collection

Before conducting the experiment, the researcher sought permission from the Division of City Schools Superintendent of Iligan City and the Principal of Iligan City East High School to conduct the study. Upon approval of the request, she asked the Guidance Office of MSU-Iligan Institute of Technology to administer the Otis-Lennon School Ability Test to the six sections of fourth year students in ICEHS. This was done to match students between experimental and control groups of the high and low ability levels. According to Leedy and Ormrod (2001), "Identifying matched pairs in the two groups is one way of strengthening the pretest-posttest control group designs". The pairings, however, were not all perfect matches. Comparison was made between the matched OLSAT scores revealed no significant difference upon application of t-test for the high ability ($t = .202$, $\rho = .841$) and low ability ($t = .536$, $\rho = .594$) groups. Comparison of matched OLSAT scores across treatment was also done. The t-value of .087 ($\rho = .931$) indicated no significant difference between the matched scores of the experimental and control groups. This implies that the pairing of students between the ability groups across treatment was comparable at the start of the study.

Once done, the two treatment conditions were randomly assigned to the two ability groups. The experimental groups (experimental-high and experimental-low) were to be exposed to the teaching of Algebra and Trigonometry with integration of history of mathematics. The control groups (control-high and control-low) were to be taught Algebra and Trigonometry the traditional way without the integration of history of mathematics.

As soon as the treatments were assigned, the researcher gave the pretest on Achievement and Affective Instruments to the four groups of students during their mathematics classes. This was done during the first week of the second semester. Two weeks prior to the conduct of the experiment, the participating teacher was introduced to the design of the experiment. She was given directions on how to implement the integration of History of Mathematics in Algebra and Trigonometry in the classroom based on the teaching plans developed by the researcher. She was also oriented on the supplemental materials made by the researcher and the necessary strategies needed to capture the interest of the students.

A simulation of the strategy of integrating history of mathematics was conducted with a week before the actual implementation of the experiment to the actual subjects. The life of Babbage was integrated in the discussion of using the calculator in finding the values of trigonometric functions. The purpose of the dry run was for the teacher to get used to the idea of integrating history in mathematics.

The students were taught how to write a journal by reading to them a sample journal. They were also provided a copy of the sample journal. Furthermore, to encourage students to write, journal writing was given a minimal weight in the computation of the students' third grading grade. Journal writing was done as an assignment. This assignment was given after discussing a particular subject matter. Although the teacher gave instructions and collected journals, the researcher checked all the journal entries. The researcher indicated individual comments in all the journals; in turn the teacher discussed these comments in a general way to the class.

Data Analysis

All the quantitative and qualitative data were analyzed to answer the questions posed in this research. For the quantitative analysis, descriptive statistical techniques were employed to present the profile of the subjects in the four groups. The t-test for paired samples was used to determine the difference in the effects of integrating history of mathematics in the instructional materials and in the teaching of mathematics on achievement in Algebra and Trigonometry compared to non-integration of the same. Two-way analysis of variance was used to determine the main effects and interaction of the treatment with ability and gender using the pretest scores as covariate. An intercorrelation matrix showed the relationship among the dependent and independent variables. Cronbach alpha was used to determine the reliability of each instrument. All hypotheses were tested using $\alpha = 0.05$. All computations were done using the SPSS program.

The qualitative data were gathered from the students' journals and the interviews conducted. Content Analysis of students' journals was also done.

Coding of Qualitative Data

In the presentation of the qualitative data, codes were used for the students. These codes indicate the group, ability, gender, and student identification number. The experimental group and control group were coded E and C, respectively. The high ability group was coded H while L was the code for the low ability group. M is the code for male subjects while F for females. Thus, the code ELM01 means "Experimental, Low ability, Male, student 1" or CHF13 means "Control, High ability, Female, student 13".

Results and Findings

Integration of History of Mathematics and Achievement

Table 2 presents the number of cases, means, and standard deviations in the pretest and posttest achievement scores by treatment, ability and gender.

Table 2. Number of Cases, Means and Standard Deviations of Scores by Treatment, Ability and Gender in the Pretest and Posttest Achievement Test

TREATMENT	Experimental (N = 59)		Control (N = 59)		Total (N = 118)	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
ABILITY						
High (N=62)						
Mean	21.06	27.23	12.22	25.35	16.64	26.29
SD	4.73	5.53	4.79	6.38	6.49	6.00
Low (N=56)						
Mean	10.25	14.61	7.50	12.07	8.87	13.34
SD	4.70	4.86	3.73	4.07	4.43	4.63
GENDER						
Male (N=48)						
Mean	17.00	22.58	11.79	18.63	14.39	20.60
SD	7.89	7.91	5.06	8.24	7.40	8.24
Female(N=70)						
Mean	15.20	20.81	8.74	19.34	11.97	19.83
SD	6.66	8.37	3.61	8.90	6.23	8.59
TOTAL (N=118)						
Mean	15.93	21.24	9.98	19.05	12.96	20.14
SD	7.18	8.19	4.90	8.58	6.81	8.43

In Table 2, the high ability experimental group consistently obtained the highest mean achievement score in both the pretest (21.06) and the posttest (27.23) while the low ability control group also consistently got the lowest mean pretest (7.50) and posttest (12.07) achievement scores.

A two-way analysis of variance controlling for the pretest mean achievement scores was done and the result is shown in Table 3.

Table 3. Two – Way Analysis of Variance of Mean Posttest Achievement Scores Controlling for the Pretest Scores by Treatment across Ability levels

Dependent Variable: POSTTEST

Source of Variation	Type III Sum of Squares	df	Mean Square	F	Sig
Corrected Model	4898.147(a)	4	1224.54	55.403	.000*
Intercept	1809.838	1	1809.84	106.444	.000*
PRETEST	470.402	1	470.402	16.996	.000*
ABILITY	726.01	1	726.01	62.589	.000*
TREATMENT	61.285	1	61.285	0.057	0.812
ABILITY * TREATMENT	1.806	1	1.806	2.81	0.096
Error	2475.084	113	22.099		
Total	51921	118			
Corrected Total	7373.231	117			

a R Squared = .662 (Adjusted R Squared = .650)

*Significant at .05 level

After adjusting for the effect of pretest, the statistical results indicate that the main effect of treatment is not significant (F = .057, p = .812). However, the main effect of ability is significant (F = 62.589, p = .000). There is no significant interaction effect between the effects of ability and treatment on mathematics achievement (F = 2.810, p = .096). This means that high ability students performed significantly better than the low ability students. Furthermore, this suggests that the performance of the experimental students did not differ significantly from the performance of the control students in both ability levels. This confirms the findings of Welch and Walberg (1972) in their report on the

Project Physics Course (a project which incorporated directly into the course substantial material from the history of science) where "no significant differences were found on the cognitive measures (achievement tests) in the course comparisons" (p. 378).

Comparison between mean gain scores of the experimental and control groups of the high and low ability levels is shown in Table 8. It can be gleaned from the table that the high ability experimental group had a gain score of 6.17 from pretest to posttest. The high ability control group, on the other hand, had a gain score of 13.13 which is twice the mean gain score of the experimental group.

Upon application of t-test for paired samples (because the students were matched) at 0.05 level, gave a t-value of -5.08 which is very highly significant. This means that the mean gain of high ability control group was significantly higher than the gain of high ability experimental group in the Achievement test.

Table 4. Comparison of the Mean Gain Scores in the Achievement Test by Treatment and Ability

Compared Groups	N	Gain	t-value	Significance
High Ability				
Experimental	31	6.17	-5.08	.001*
Control	31	13.13		
Low Ability				
Experimental	28	4.36	-0.138	0.891
Control	28	4.57		
Total				
Experimental	59	5.31	-3.211	.002*
Control	59	9.07		

* Significant at 0.05 level

This result conforms to the findings of Chi-Kai Lit et al. (2000) in their study on "The Use of History in the Teaching of Mathematics" wherein "the difference in the mean achievement scores of the experimental and control groups were found to be statistically significant in favor of the control group" (p. 43). This implies that the test scores of the students in the experimental group were lower than those in the control group. The gain scores of both the low ability experimental and low ability control groups were almost the same, that is, 4.36 and 4.57 respectively. The computed t-value of -0.138 indicates no significant difference between the mean achievement scores of the two groups. The experimental low and control low groups performed equally.

A two-way analysis of variance controlling for the pretest mean achievement scores of males and females in the experimental and control groups was done. The result is shown in Table 5. After adjusting for the effect of the pretest, the statistical results signify that that the main effect of treatment is not significant (F = 1.499, ρ = .223). The main effect of gender, however, is significant with F-value of 5.063 (ρ = .026). There is no significant interaction effect between the effects of ability and treatment on achievement in mathematics (F = 3.081, ρ = .082). This means that males and females in either the experimental or control groups did not perform differently.

Table 5. Two – Way Analysis of Variance of the Mean Posttest Achievement Scores Controlling for Pretest by Treatment and Within Gender Groups

Dependent Variable: POSTTEST

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4208.416(a)	4	1052.104	24.214	.000*
Intercept	1144.129	1	1144.129	32.195	.000*
PRETEST	3777.732	1	3777.732	91.255	.000*
TREATMENT	0.341	1	0.341	1.499	0.223
GENDER	40.968	1	40.968	5.063	0.026
TREATMENT * GENDER	5.74E-02	1	5.74E-02	3.081	0.082
Error	3164.815	113	28.257		
Total	51921	118			
Corrected Total	7373.231	117			

a R Squared = .571 (Adjusted R Squared = .555) * Significant at .05 level

Table 6 shows the comparison of the mean gain scores by treatment and gender. The males in experimental group (5.58) and in the control group (6.84) did not differ in the gain score ($t = -0.82$, $\rho = 0.416$). They gained the same way from pretest to posttest. This observation, however, is not true of the females. The difference in the mean gain scores of the females in the experimental vs. the control group was very significant ($\rho < 0.001$). The female control group performed significantly better than the female experimental group in the achievement test.

The significant difference between the control group and experimental group (combined male and female) is accounted for largely by the difference between the performances of the females, not the males. In the course of the experimentation, the students' and teacher's main feedback on the use of the History of Mathematics lessons in Algebra and Trigonometry was "lack of time" to finish the lessons. This observation was noted by the observer when she wrote:

The low ability experimental group's class began at 8:50 and supposed to end at 9:30, but the math class had to extend up to 9:45 making use of the recess time just to finish the lesson. On the other hand, the high ability experimental group started math class at 4:20 in the afternoon and ended at 5:20, the earliest. The afternoon class is supposed to end at 5:00 in the afternoon. (Nov. 28, 2001)

Table 6. Comparison of the Achievement Mean Gain Scores by Treatment and Gender

Compared Groups	N	Gain	t-value	Significance
Male				
Experimental	24	5.58	-0.82	0.416
Control	24	6.84		
Female				
Experimental	35	5.11	-0.334	.001*
Control	35	10.6		
Total				
Experimental	59	5.31	-3.211	.002*
Control	59	9.07		

* Significant at 0.05 level

During some informal discussions with the teacher, she hinted that the experimental groups (both the high and the low ability groups) were behind in their lessons. She said this was one difficulty she encountered in the implementation of the lessons. This was the concern of the teacher in her comments:

"Some students feel there was not very much time left for actual math lessons to learn, the 40-minute period is not enough."

This comment of the teacher was reinforced by one of her student.

"Integration of History of Math is just fine to discuss but I just find that sometimes we spend more time on it". (EHF09)

The teacher tried to catch up. A strategy she used was suggested in the following comments:

I feel that it is consuming much time to follow all the historical topics as outlined in the lessons. There were occasions that I just gave the gist of the history rather than have a lengthy discussion of it. Also I give assignments on the topics of history of math to compensate for the lack of time.

Although the teacher and some students perceived that a portion of the time spent in discussing mathematics lessons was spent in history discussions, still, they were able to finish discussing all the lessons. However, this situation was not beneficial to the low ability students. This was mentioned in one of the informal interviews of one student who belongs to the low ability level:

"Sa ako, Ma'am, bisan ug gamay ra ang among nahuman nga leksyon basta kami tanan makasabut. Seguro mas maayo pa kana kaysa mahuman namo ang tanan pero wala diay mi nasabtan." [For me, Ma'am, even though we take up only a few topics as long as we understand the lesson. That's better than we cover up the whole lessons without any understanding] (ELMH11)

One argument against deliberately integrating history to the teaching of mathematics is that history discussions take time from regular classroom study of mathematics. In the present study, three students in the high ability group and one in the low ability group perceived that the integration of history helped them to understand Algebra and Trigonometry lessons easily. The following excerpts are from their essays:

> *"I do appreciate this integration of history in our math lesson because it helps me understand more about the lesson"* (EHF13)
>
> *"It helped me to be positive in understanding Algebra and Trigonometry"* (EHM10)
>
> *"You learn to view math at another angle and that's how you understand it."* (EHF15)
>
> *"It was my first time to go to our library just to research about Descartes and this helped me understand the lesson on plotting points easily."* (ELF07)

Student ELF07 had hinted from her essay that the method of integrating history in Algebra and Trigonometry motivated her to go further by visiting the library.

From observations, it was not really the history discussions in the lessons that caused the experimental group to lag behind in the lesson but the teacher's inadequate skills in leading the discussion. There were times when the discussion went beyond the main topic. When student showed enthusiasm and interest in the historical issues, the teacher was carried away. Her attention was called as to the probable results of some lengthy historical discussions. However, having used the approach for the first time, she needed more training and experience to gain expertise.

The analysis of the achievement scores indicated that the time spent in the integration of history in the teaching of mathematics did matter in the low ability group. It appears that the low ability students needed more time to grasp mathematical concepts and processes and the time taken for the integration of history to mathematics teaching affected their performance.

The non significant difference in the performance of students from the high ability level both in the experimental and control groups was further studied. The 118 students' scores of the MSU-Systems Admission School Examination (MSU-SASE) from the MSU-IIT admissions Office is presented in Table 7. The MSU-SASE is usually taken by all entering qualified freshmen college students from all over Mindanao, Sulu and Palawan regions who wish to enroll at any school in the MSU System. MSU-SASE is a validated and reliable test. The test includes Language Usage, Science, Mathematics, and Aptitude questions. The Language Usage (LU) part of the test measures the reading comprehension ability of a student. An interview with Prof. Luisa Mama-o of the Admissions Office of MSU-IIT on March 24, 2002 confirms the purpose of the Language Usage part of the test. She said:

> *"A student who passes the test, that is, who correctly answers at least 40 of the 80 items is considered good in reading comprehension. However, if a student fails in this test, that is if he gets a score below forty, he is considered poor in reading comprehension"*

Of the 118 students included in the study, only 32 or 27.12% passed the Language Usage examination while 61% or 72 students failed the examination. This means that more than fifty percent of the subjects of the study had poor reading comprehension and 12% of these were the experimental-high group; another 12% were in the control high group. Thirty seven (37) percent of the low ability group had poor reading comprehension. The relationship between the MSU-SASE Language Usage Score and the Achievement Test Score was further examined using Pearson's Product Moment Correlation r.

Table 7. Frequency Distribution of MSU-SASE Language Usage Scores Obtained by the 118 Samples included in the Study

MSU-SASE LU Scores	40 items and more		Less than 40 items		Did not take the exam.		Total	
	N	%	N	%	N	%	N	%
High Ability								
Experimental	17	14.41	14	11.86	0	0	31	26.27
Control	14	11.86	14	11.86	2	1.69	31	26.27
Low Ability								
Experimental	1	4.75	22	18.64	5	4.24	28	23.73
Control	0	0	21	17.80	7	5.93	28	23.72
Total	32	27.12	72	61.02	14	11.86	118	100.00

The MSU-SASE Language Usage scores and the Achievement scores of both the experimental ($r = .65$, $\rho = .001$) and control ($r = .79$, $\rho = .001$) groups were positively and significantly related. This means that the higher the score of the student in the achievement test, the better is his/her reading comprehension; the lower is his achievement score, the poorer is his reading comprehension. This implies further that at least fifty percent of the students included in the study had poor reading comprehension. An interview with the teacher handling the classes in the experiment confirmed this result. She said: "*Most of the fourth year students, especially those belonging to the last two sections, could not read properly.*"

Attitude towards Mathematics The Attitude-Towards-Mathematics Inventory consisted of 13 items constructed by the researcher and 15 selected items from Morco's (1994) Attitudinaire. The items used a 5-point response scale. The highest possible score in the Inventory is 140; the lowest, 28. The higher the score is, the more favorable is the attitude of the respondents.

Table 8 shows that the high ability students in the experimental group had the highest mean (105.39) while the low ability control group had the lowest mean (95.89) in the pretest.

Table 8. Number of Cases, Means and Standard Deviations of the Pretest and Posttest Scores in the Attitude-Towards-Mathematics' Inventory by Treatment, Ability and Gender

TREATMENT	Experimental (N = 59)		Control (N = 59)		Total (N = 118)	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
ABILITY						
High (N=62)						
Mean	105.39	107.23	104.97	106.06	106.18	106.65
SD	10.47	9.32	10.75	9.02	10.53	9.11
Low (N=56)						
Mean	97. 89	96.04	95.89	96.00	96.69	96.02
SD	11.49	16.19	11.89	12.45	11.62	14.31
GENDER						
Male (N=48)						
Mean	97.17	100.13	96.79	101.17	96.98	100.65
SD	12.52	8.79	14.51	12.79	13.41	10.88
Female(N=70)						
Mean	105.03	103.14	103.31	101.37	104.17	102.26
SD	9.69	16.82	9.47	11.31	9.56	14.26
TOTAL (N=118)						
Mean	101.83	101.92	100.66	101.29	101.25	101.60
SD	11.51	14.10	12.11	11.83	11.77	12.96

The attitude-towards-mathematics scores of the experimental and control groups across the ability levels improved after the experiment, except that of the low ability experimental group whose score decreased by almost two points. A closer look at the values of the standard deviation reveals that except for the low ability experimental group, all the three groups were more or less homogenous in their scores in the post treatment administration of the Mathematics Attitude Inventory.

The female students in the experimental group consistently scored highest in attitude in both the pretest (105.03) and the posttest (103.14). On the other hand, the male students in the control group scored lowest in the pretest (96.79) while the male students in the experimental group had the lowest posttest mean score (100.13). The scores of the males improved from pretest to posttest in both treatments. This is not the case among the females. The standard deviations of the males and the females in the experimental and control groups indicated that the scores of the two groups vary from 2 to 5 points.

Table 9. Comparison between the Posttest Attitude-Towards-Mathematics Scores by Treatment and Ability

Compared Groups	N	Posttest Scores	Difference	t-value	Probability level	Significance
High Ability						
Experimental	31	107.23	1.17	0.498	0.62	NS
Control	31	106.06				
Low Ability						
Experimental	28	96.04	0.04	0.009	0.993	NS
Control	28	96				
Total						
Experimental	59	101.92	0.63	0.61	0.821	NS
Control	59	101.29				

NS - not significant

A comparison of the posttest attitude-towards-mathematics mean scores is presented in Table 9. The high ability experimental group had a slightly higher mean score (107.23) than the high ability control group (106.06). The difference (1.17) was found to be not significant (t = 0.498, ρ = 0.62). This means that the high ability students across treatment did not differ in their attitude-towards-mathematics. The same observation is true of the low ability group. No difference was found between the posttest scores of the low ability students in the experimental (96.04) and control (96.00) groups.

A two-way analysis of variance was computed for determining the effect of treatment and ability on attitude–towards–mathematics mean scores. The statistical results indicate no significant main effect (F = 0.116, ρ = 0.734) of treatment on the attitude. The interaction effect of treatment and ability shows no significant effect (F = 0.116, ρ = 0.734) on attitude either. This suggests that the attitude-towards-mathematics scores of the experimental students did not differ significantly from the attitude-towards-mathematics scores of the control students.

High ability students in both treatments scored higher in the Attitude-towards-mathematics Inventory than the low ability students (Table 9). The statistical results of the two-way ANOVA had also shown that the main effect of ability on attitude-towards-mathematics scores is very significant with F-value of 14.37 and an associated probability of 0.001.

Table 10. Comparison of the Posttest Attitude-Towards-Mathematics Scores by Treatment and Gender

Compared Groups	N	Posttest Scores	Difference	t-value	Significance
Males					
Experimental	24	100.13	-1.04	0.329	NS
Control	24	101.17			
Females					
Experimental	35	103.14	1.77	0.517	NS
Control	35	101.37			
Total					
Experimental	59	101.92	0.63	0.61	NS
Control	59	101.29			

NS - not significant

This suggests that high ability students have more positive attitude towards mathematics than the low ability students. This result agrees with the findings of Renner, Abraham, and Stafford (1978) in their summary of studies on attitudes toward science from 1976, where they reported that "higher achieving students have better attitudes toward science than lower achieving students" (p.66).

A comparison of the posttest attitude-towards-mathematics scores by treatment and gender is presented in Table 10. The paired samples t-test of difference between the posttest attitude-towards-mathematics scores of the experimental and those of the control groups across gender yielded no significant difference.

Some excerpts from several students' compositions presented below provide an idea of how integration of history of mathematics affected their attitude towards and views about mathematics.

> *"The lessons containing history of math really change my attitude and views about math. Before I thought that math is so boring 'cause* [in] *all my math class from first year up to now, we merely do solving that don't bring amusement or please us. Like, your math teacher gives you a problem then afterwards everybody of the class made correct answers after that wala na* (no more). *But now, I like math because of the wise quotations and funny anecdotes we discuss in class."* (EHM07)

> *"I find it hard to find the solutions to the complicated formula of Surya Siddhanta. After solving, I am happy when teacher B. told me my answer is correct. Maayo gani karon kay naa nay calculator.*(It's good that we have calculators now). *I was just thinking how the mathematicians have made the formula of Table SS without a calculator. This has changed my bad attitude towards math.* (EHF09)

> *"The lessons have improved my attitude toward mathematics; showed me how creative mathematicians were."* (EHF01)

> *"Because of this, it enhanced all my positive attitudes towards math and views about mathematics."* (EHM13)

> *"Naganahan ko atong naay history ang math kay nadugangan man ang akong paglantaw sa math. Dili lang diay sige ug compute pero gahisgot pud diay ug kinabuhi sa mga tawo sama nato nga ordinaryo pero kahibalo mo discover ug math.* (I like math with its history because it enriches my views about math; that math is not always dealing with computations but discusses the lives of mathematicians who are just ordinary persons like us.) (ELF06).

The foregoing comments are representative of the general feeling of the group. Not one student gave remarks contrary to those presented above. The students reported that the integration of history of mathematics in Algebra and Trigonometry changed their attitudes toward mathematics as well as their views about the subject. The result of the qualitative analysis on attitude confirms the findings of Anota (1981) that the use of affect-oriented mathematics lessons effected a more favorable attitude towards mathematics among students.

Not only the students but also the participating teacher noted the positive effects of integration of history of mathematics in Algebra and Trigonometry. The teacher wrote in her observation notes:

> *"I did not observe a big difference in their performance but I have seen a big difference in their attitudes towards mathematics. Students began to appreciate the beauty of mathematics: that it is not something to be feared. The students became very participative in class discussions. In fact our librarian reported to me that my students frequently go to the library just to read about the lives of the mathematicians that I have assigned to them. The historical part helped a lot."* (Dec. 18, 2001).

Perceived Usefulness of Mathematics

Table 11 shows the means and standard deviations in the pretest and posttest scores on the students' perceived usefulness of mathematics. The mean scores of the groups are shown by treatment, ability and gender. The highest possible score in the Perception of the Usefulness of Mathematics Scale is 50.

Table 11. Means and Standard Deviations of the Rated Perceived Usefulness of Mathematics Before and After the Treatment by Ability and Gender

TREATMENT	Experimental (N = 59)		Control (N = 59)		Total (N = 118)	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
ABILITY						
High (N=62)						
Mean	29.52	40.52	36.55	37.77	33.03	39.15
SD	2.64	3.84	5.27	5.93	5.45	5.14
Low (N=56)						
Mean	35.21	33.54	33.46	33.68	34.34	33.61
SD	5.19	4.21	4.37	5.58	4.83	4.90
GENDER						
Male (N=48)						
Mean	31.54	36.46	34.38	36.04	32.96	36.25
SD	4.39	5.48	5.03	6.05	4.89	5.71
Female(N=70)						
Mean	32.69	37.71	35.57	35.69	34.13	36.70
SD	5.29	5.22	5.11	6.18	5.36	5.77
TOTAL (N=118)						
Mean	32.22	37.20	35.08	35.83	33.65	36.62
SD	4.94	5.31	5.07	6.08	5.18	5.73

The high ability respondents in the control group registered the most favorable perception of the usefulness of mathematics in the pretest (36.55) while the high ability experimental group had the lowest pretest mean score (29.52). However, in the posttest, the high ability students in the experimental group scored highest (40.52) and the low ability in the control group scored lowest (33.68). The scores in all the four groups were homogenous as evidenced by the small standard deviations.

The females in the control group (Table 11) scored the highest in the pretest (35.57), the males in the experimental group scored the lowest (31.54). On the other hand, the females in the experimental group obtained the highest score in the posttest (37.71); those in the control group, the lowest posttest mean (35.69). All the groups showed improvement in their perception of the usefulness of mathematics in the posttest.

Table 12. Two – Way Analysis of Variance of the Posttest Scores Controlling for the Pretest Scores for Perceived Usefulness of Mathematics by Treatment and Ability

Dependent Variable: POSTTEST

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1361.634(a)	4	340.409	15.537	.000*
Intercept	1078.252	1	1078.252	49.213	.000*
PRETEST	342.401	1	342.401	15.626	.000*
TREATMENT	1051.698	1	1051.698	48.001	.000*
ABILITY	145.556	1	145.556	6.643	.011*
TREATMENT * ABILITY	232.726	1	232.726	10.622	.001*
Error	2475.832	113	21.910		
Total	161189	118			
Corrected Total	3837.466	117			

a R Squared = .355 (Adjusted R Squared = .332)

* Significant at .05 level

The two-way analysis of variance (Table 12) indicated that the experimental group mean is significantly higher than the control group's mean (40.52 vs. 37.77) across ability levels. The high ability and low ability groups in the two treatments scored differently ($F = 48.001$, $\rho = .000$) in favor of the high ability samples. The F-value of 10.622 and an associated probability of 0.001 indicate significant interaction effect between treatment and ability on the mean perceived usefulness of mathematics. The high and low mathematical ability students in the experimental and control groups perceived the usefulness of mathematics differently; the experimental group perceived mathematics more useful than did the control group. This result is surprising: both the high and low ability students in the experimental group were exposed to real-life problems, stories and anecdotes about the lives of mathematicians. However, it was only the high ability experimental students who attributed their positive perception of the usefulness of mathematics to the instructional materials and to the integration of history of mathematics in their lessons; the low ability students in the experimental group did not feel the same way.

Table 13. Comparison of the Posttest Mean Scores on Perceived Usefulness of Mathematics by Treatment and Gender

Groups Compared	N	Posttest Mean	Difference bet. means	t-value	Significance
Male					
Experimental	24	36.46	0.42	0.25	NS
Control	24	36.04			
Female					
Experimental	35	37.71	2.02	1.484	NS
Control	35	35.69			
Total					
Experimental	59	37.2	1.37	1.319	NS
Control	59	35.83			

NS - not significant

The male and female students in the experimental group have slightly higher mean perception score than their counterpart in the control group. A comparison of the perceptions of usefulness of mathematics of males and females in the two treatment groups (Table 13) showed no significant gender differences between the experimental and control groups' perceptions, implying that students have the same perceptions of the usefulness of mathematics, irrespective of the way the subject was taught.

The male and female students in the experimental group have slightly higher mean perception score than their counterpart in the control group. A comparison of the perceptions of usefulness of mathematics of males and females in the two treatment groups (Table 13) showed no significant gender differences between the experimental and control groups' perceptions, implying that students have the same perceptions of the usefulness of mathematics, irrespective of the way the subject was taught.

Qualitatively, the high ability students in the experimental group perceived mathematics more useful than their counterparts in the control group. They attributed their positive perception of the usefulness of mathematics to the integration of history of mathematics in the lessons. Consider the following excerpts from the students' journals:

> *"I like the addition of history to our math subject. I can say how necessary it is to study mathematics because without it, the world won't be progressive"* (EHF09)
>
> *"To listen and read history of mathematics is very interesting. It shows how important mathematics is to me personally.* "(EHM10)
>
> *"In the story of the Frustrated Tour Guide, Thales was able to solve for the height of the pyramid using mathematics. This made me learn more about the value of math."* (EHF16)
>
> *"I need to study math because it is useful and I might need it for my future job."* (EHF07)
>
> *"My perceptions of the usefulness of mathematics are mostly good. Good because many of the lessons we have on history of math started with simple thing like a fly and then we have the Cartesian plane."* (EHM03)
>
> *Very useful,* [in] *everything you see there is mathematics even in plants, insects, everything."* (EHM12)
>
> *I can definitely say that the lessons* [history] *are truly applicable to real life situations.* "(EHF18)

The comments above are representative of how the high ability students perceive the usefulness of mathematics. The participating teacher also had favorable comments in her observation notes regarding this:

> "*The lessons* [in history of mathematics] *had given the students reasons to perceive math useful in their lives. Michael who belongs to the high ability experimental group perceived that math is useful. He offhandedly mentioned while we were walking towards the classroom that math is really useful because through math we have good bridges and roads, tall and great buildings, etc.*" (Dec. 5, 2001)

Confidence in Learning Mathematics The means and standard deviations of the self-rated confidence-in-learning-mathematics scores in the pretest and posttest scores by treatment, ability and gender are shown in Table 14. Fifty (50) is the highest possible score in this Scale.

Table 14. Mean Gains for Self-rated Confidence-in-Learning-Mathematics by Treatment, Ability and Gender

TREATMENT	Experimental (N = 59)			Control (N = 59)			t-ratio	Sig.
	Pretest	Posttest	Gain	Pretest	Posttest	Gain		
ABILITY								
High (N=62)								
Mean	31.65	31.32	-0.33	29.81	31.61	1.80	1.89	NS
SD	3.86	3.16		3.97	3.78			
Low (N=56)								
Mean	29.93	30.39	0.46	30.46	30.14	-0.32	0.75	NS
SD	2.72	3.18		2.12	3.37			
GENDER								
Male(N=48)								
Mean	31.38	31.04	-0.34	30.75	31.75	1.00	0.99	NS
SD	3.94	2.71		2.91	3.99			
Female(N=70)								
Mean	30.46	30.77	0.31	29.69	30.34	0.65	0.36	NS
SD	3.07	3.49		3.38	3.32			

NS – Not Significant

The high ability respondents in the experimental group obtained the highest mean (31.65) while its counterpart in the control group had the lowest mean (29.81) score for confidence-in-learning-mathematics before the treatment. After the treatment, the high ability students of the control group scored highest (31.61) while the control low ability group scored lowest (30.14). It is apparent from Table 14 that scores on confidence-in-learning-mathematics are similar across ability and treatment groups except for the SD-value (2.12) of the control low ability group in the pretest.

Table 15. Comparison of the Post Treatment Confidence in Learning Mathematics Scores by Treatment, Ability and Gender

Compared Groups	N	Posttest Scores	Difference	t-value	Significance
High Ability					
Experimental	31	31.32	-0.29	0.328	NS
Control	31	31.61			
Low Ability					
Experimental	28	30.39	0.25	0.285	NS
Control	28	30.14			
Male					
Experimental	24	31.04	-0.71	0.719	NS
Control	24	31.75			
Female					
Experimental	35	30.77	0.43	0.527	NS
Control	35	30.34			
Total					
Experimental	59	30.88	0.99	0.305	NS
Control	59	29.89			

NS – not significant

The comparison of the mean gain scores between the experimental and control groups by ability level are also shown in Table 14. The application of t-test for paired samples on the mean gain score of the experimental high (-0.33) and control high ((1.80) showed a non significant difference (t = 1.89, ρ = .06). The mean gain score of the experimental low (0.46) was also compared with the control low's mean gain score (-0.32) providing no significant result (t = 0.75, ρ = .46). Moreover, comparison on the mean gain scores of the experimental and control groups across gender was also done and revealed no significant results on either males (-0.34 vs. 1.0; t = 0.99, ρ = 0.33) or females (0.31 vs. 0.65; t = 0.36, ρ = 0.72). Across treatment, ability and gender, the students rated themselves in the same way on the Confidence in Learning Mathematics Scale.

The difference in the posttest scores of the experimental (31.32) and control (31.61) groups in the high ability level was shown to be not significant with t-value of 0.328 for paired samples and an associated probability of .774 as presented in Table 15. Similarly, the comparison between the posttest scores of the experimental low (30.39) and control low (30.14) respondents yielded no significant difference upon application of t-test for paired samples (t = 0.285, ρ = .776).

The comparison of the posttest achievement scores by treatment and gender likewise showed no difference between males from the experimental (31.04) and control (31.75) groups (t = 0.719, ρ = .476).

Males and females have the same confidence showing that the effect of integrating history of mathematics did not differ for these groups. Likewise, the experimental and control groups did not differ in their confidence. This shows that integrating history of mathematics in Algebra and Trigonometry does not affect the confidence in learning mathematics. Both the experimental and control groups have positive slight increase in their mean confidence scores from pretest to posttest. However, these slight increases are not significant.

A content analysis of the students' compositions reveals these common feelings about mathematics: *difficult, challenging, boring, demanding.* One report reads:

"Most of us feel it challenging but Arnel feels that it is going to be hard. Luigi and Al Azanereeh are nervous because Algebra and Trigonometry is difficult subject but all of us are agreed on trying our best." (EHM01)

Several students reported that they *enjoy doing mathematical activities and computations* but they *do not like solving word problems because these are difficult*. Students associate mathematics with computational skills. At the start, they were already pessimistic about algebra and problem solving.

Students in the experimental group admitted that they had a hard time doing mathematics. Did integration of mathematics history help change their views about mathematics?

Consider the following students' comments:

"There is a great effect on my personality upon listening and reading the history of math. It challenges me to know that mathematicians could also commit mistakes." (EHF02)

"Having this kind of lessons affect me personally. It has helped me motivating myself to learn mathematics. Our lessons discuss the appreciation of what the mathematicians did for mathematics." (EHM07)

"It amazed me to realize that some math concepts were discovered by mere observations, persistence and play." (EHF04)

"Our lessons in history of math affected me personally on the way I see things. I have developed this attitude of thinking ahead. I kind of like this attitude it save most of my time. Every time I see or read something about history of math I kind of associate it with the concept of math that it represents and the lessons it give me." (EHF08)

"The lessons make me realize that there is a simple solution to a problem. You first have to find it". (EHM04)

Students in the control group, on the other hand, admitted that doing mathematics is difficult, however, they declared:

"About the trigonometric function, well, it's really a challenging lesson. Yes, it's difficult, but I'm having fun in trying to solve the problems. It's really fun especially solving it with my friends and my classmates. It's fun to make arguments with others in this lesson." (CHF13)

"Math is difficult and needs understanding of its topics but it's really challenging to solve problems as what my classmate named Joan told me. Sometimes I decided to give up solving but her words persuaded me to continue and learn a lot from it." (CHF09)

"I like the topic on exponential function. It doesn't mean that I have understood the lesson. It is not that. I see to it that if ever I'm confused with some problems, I will ask someone who knows that would teach me. I don't want to go home with confused head." (CHF12)

In a research designed to study the relationship between confidence in ability and academic performance of over 1000 students from the time they started seventh grade until they had completed tenth grade, Brookover and his associates (1974, as reported by Lindgren, 1976) found that confidence was a significant factor in achievement at each grade level studied. The study revealed that "although a significant proportion of students with high confidence achieved at a relatively lower level; practically none of these students with lower (less positive) confidence achieved at a high level". The present study confirms the earlier study that low ability students have lower confidence than high ability students.

Anxiety in Mathematics The highest possible score in the Mathematics Anxiety Scale is 50 while the lowest is 10. The means, standard deviations and comparison of the pretest and posttest anxiety in mathematics scores of the samples classified by treatment, ability and gender are summarized in Table 16.

The control high ability group obtained the highest mean in the pretest mathematics anxiety score (30.23) while the experimental high ability group had the highest mean posttest score for mathematics anxiety (31.26). The control low ability group consistently obtained the lowest mean mathematics anxiety score in the pretest (28.68) and the posttest (29.86).

Table 16. The Mean Gain Scores on Anxiety in Mathematics of the Samples Compared Before and After Treatment by Ability and Gender

TREATMENT	Experimental (N = 59)			Control (N = 59)			t-Ratio	Sig.
	Pretest	Posttest	Gain	Pretest	Posttest	Gain		
ABILITY								
High (N=62)								
Mean	29.06	31.26	2.20	30.23	30.97	0.74	1.16	NS
SD	5.07	4.71		5.16	4.38			
Low (N=56)								
Mean	28.71	29.89	1.18	28.68	29.86	1.13	-0.8	NS
SD	3.29	3.95		3.30	3.97			
GENDER								
Male (N=48)								
Mean	29.63	30.75	1.12	30.13	31.00	1.31	0.19	NS
SD	3.71	4.11		3.19	3.82			
Female(N=70)								
Mean	28.40	29.71	0.87	29.06	30.06	1.0	0.26	NS
SD	4.65	4.74		5.08	4.44			

NS Not Significant

The highest pretest and posttest scores were consistently achieved by the males in the control group (30.13 and 31.00, respectively) whereas the females of the experimental group consistently had the lowest pretest (28.40) and posttest (29.71) scores. The pretest scores vary while the posttest scores in all the four groups were homogenous as evidenced by the standard deviations.

The mean gain scores of the experimental (2.2) and control (0.74) groups in the high ability level do not differ significantly (t = 1.16, ρ = .25) as shown in Table 16. The same is true of the low ability experimental and control groups (1.18 vs. 1.13; t = -.8, ρ = .43); the males of the experimental (1.12) and control (1.31) groups (t = .19, ρ = .85) and the females (.87 vs. 1; t = .26, ρ = .8). All the eight groups performed similarly in the Mathematics Anxiety Scale.

Table 17. Comparisons of the Experimental and Control Groups' Posttest Mathematics Anxiety Scores according to Ability and Gender

Compared Groups	N	Posttest Scores	Difference	t-value	Significance
High Ability					
Experimental	31	31.26	0.29	0.251	NS
Control	31	30.97			
Low Ability					
Experimental	28	28.89	-0.97	0.911	NS
Control	28	29.86			
Male					
Experimental	24	30.75	-0.25	0.218	NS
Control	24	31			
Female					
Experimental	35	29.71	-0.35	0.312	NS
Control	35	30.06			
Total					
Experimental	59	30.14	-0.3	0.876	NS
Control	59	30.44			

NS - Not Significant

The difference in posttest mathematics anxiety mean scores (Table 17) of high ability students (0.29) by treatment was found not significant ($t = .251$, $\rho = .802$). In like manner, the slight difference in the posttest scores of the low ability students from experimental (28.89) and control (29.86) were not significantly different ($t = 0.911$, $\rho = .366$). No significant differences were found either in the comparison across treatment ($t = 0.218$, $\rho = .828$) ; neither between males nor females within treatment groups. The experimental and control groups of the high ability level included in the study scored nondifferently in the Mathematics Anxiety Scale, indicating that they have the same anxiety about mathematics at the start and towards the end of the study. The same trend is also true among the experimental and control groups of the low ability level. This can also be observed among the males and females of the experimental and control groups.

The anxiety state of the control group from the high ability level was revealed in these passages from the students' journals:

> *"I find it so hard to understand the part of exponential function. Honestly speaking, I find math so hard. I already do my best to analyze all the problems that has been given to us but still my mind find it so hard to understand it and I easily forget all the lesson for that day. But I need to improve in my math this year."* (CHF01)
>
> *"This is my biggest problem in my life that sometimes I cannot understand some topics in my math subject. The topic I cannot understand well is Activity 6. And the topic I got easy is Activity 2. I try my best to study very well in Math so that I got high score. I promise to listen to my teacher as she discussed to us. So that I cannot always ask question to my classmates, and to get easy in Math subject."* (CHM12)

An excerpt from the journal of a student from the low ability control group reflects her anxiety in mathematics in a form of a prayer:

> *"Lord, nganong bugo man gyud ko? Dili lagi ko kasabut sa leksyun namo sa Math. Ambot oy, wala man gyud koy common sense or utok bolinao nalang siguro ang nabilin? Please Lord, help me intawon kay dili gyud ko makasabut. Hinaut pa unta, Lord, nga sa sunod makasabut na unta ko, Lord, help me ug hinaut pa unta nga imong dunggon akong pag-ampo, Lord, aron maka-graduate ko.* [Lord why am I so dull? I could not understand our lessons in math. Why is it that I don't have common sense or is it just the brain of 'bolinao' that is left in me? Please Lord help me because I cannot understand. I hope, Lord, that I will understand so that I can graduate.]" (CLF05)

An interesting thing to consider here is that the high ability students in the experimental and control groups were anxious and yet their achievement scores were high. Apparently, some degree of anxiety is needed for learning to take place. Some research indicates that anxiety acts as a drive that stimulates learning. As reported by Lindgren (1976), F.N. Cox administered tests of anxiety to fifth-grade boys in Melbourne, Australia and divided them into high, middle, and low anxiety. "The middle anxiety group's academic performance was significantly better than that of the other two groups." (p.230)

What seems to stimulate most effective learning is anxiety in the middle ranges. In spite of the fact that the students found their lessons difficult, the very high level of anxiety did not interfere with the learning of complex materials.

Here are some comments from the high ability students:

> *"Because math was one of my feared subjects that is why before the exam or tests in math, I conditioned myself and try to study and understand the lessons."* (EHM08)
>
> *"Sometimes it made me feel anxious because I easily lose hope when I can't get the solution. So I make it a great challenge for me to study hard in math".* (EHF10)

Consider the following comments of low ability students in the experimental group when asked about the effect of integrating history of mathematics to them personally.

> *"Good effects. Before, I cannot face the truth when I had a low grade. At least I've learned a lot from the lessons."* (ELF14)
>
> *"Kung makakuha gani ko ug gamay nga grado, maguol gyud ko pero sa dihang na'ay history nalipay ko kay dili na man ko ma-zero.* [If I get low grades, I am discouraged but when history was included I was happy because I no longer get zero.] (ELM03)

Beliefs about Mathematics Comparisons of the mean gain scores on beliefs towards mathematics classified by treatment, ability and gender is obtainable in Table 18. The lowest score for the Beliefs towards Mathematics Inventory is 14 and the highest possible score is 70.

The high ability students of the experimental group had a slightly higher pretest (52.65) and posttest (51.81) mean scores than the control group (Table 22). The experimental low group had slightly lower pretest score (48.82) but slightly higher posttest score (50.32) when compared with the control group. The experimental group males consistently obtained the highest scores for beliefs towards mathematics mean score in the pretest (51.75) and posttest (51.25) whereas males in the control group got the lowest pretest score (49.04) and control females scored lowest in the posttest (49.31). A slight decrease in scores from pretest to posttest is the general trend displayed in the table.

Table 18. Comparisons of Scores in the Pretest and Posttest on Beliefs toward Mathematics of the Experimental and Control Groups by Ability and Gender

TREATMENT	Experimental (N = 59)			Control (N = 59)			t-ratio	Sig.
	Pretest	Posttest	Gain	Pretest	Posttest	Gain		
ABILITY								
High (N=62)								
Mean	52.65	51.81	-0.84	51.97	51.48	-0.69	-0.29	NS
SD	4.21	3.82		4.26	6.64			
Low (N=56)								
Mean	48.82	50.32	1.50	49.16	47.21	-1.97	1.01	NS
SD	6.22	4.86		8.23	7.10			
GENDER								
Male (N=48)								
Mean	51.75	51.25	-0.50	49.04	49.67	0.63	-0.6	NS
SD	5.11	3.79		6.20	5.95			
Female(N=70)								
Mean	51.00	51.00	0	51.74	49.31	-2.43	1.35	NS
SD	5.65	4.78		6.65	7.93			

NS – Not Significant

The mean gain score (Table 18) of the experimental group was compared with the control group's mean gain score. The t-test for paired samples showed no significant differences between treatments among the high ability groups (-0.84 vs. -0.69; t = -0.29, ρ = 0.77); the low ability groups (1.5 vs. -1.97; t = 1.01, ρ = 0.32); the males (-0.5 vs. 0.63; t = 0.6, ρ = 0.55) and the females (0 vs. -2.43; t = 1.35, ρ = 0.18). The experimental high group and the control high group scored in the same way. This is also true with the experimental low and the control low groups, the experimental and control males as well as the experimental and control females.

Table 19. Comparison of Posttest Mean Beliefs Toward Mathematics Scores of the Experimental and Control Groups by Ability Level and Gender

Compared Groups	N	Posttest Scores	Difference	t-value	Significance
High Ability					
Experimental	31	51.81	0.33	0.234	NS
Control	31	51.48			
Low Ability					
Experimental	28	50.32	3.11	1.911	NS
Control	28	47.21			
Male					
Experimental	24	51.25	1.58	1.099	NS
Control	24	49.67			
Female					
Experimental	35	51	1.69	1.08	NS
Control	35	49.31			
Total					
Experimental	59	51.1	1.64	1.074	NS
Control	59	49.46			

NS - not significant

The mean posttest beliefs scores, in Table 19, when compared across treatment and ability levels do not differ significantly for high ability group (t = .234, ρ = .816). This is also true between the low ability groups in the experimental and the control groups (t = 1.911, ρ = .061). The experimental and control groups of the high and low ability levels did not differ in their posttest beliefs scores. The high ability experimental and control groups scored similarly in the Beliefs toward Mathematics Inventory so with the low ability experimental and control groups.

Table 19 also presents the comparison of posttest beliefs scores classified according to treatment and gender. The males in the experimental and control groups performed nondifferently in the Beliefs Inventory. The females in the experimental group, on the other hand, scored in the same way as the control female group.

A study of the compositions made by high ability students reveals positive beliefs of the students regarding the effect of history of mathematics on them.

> *"I thought that math is a difficult subject and not interesting. The lessons in history of math had changed my beliefs."* (EHF05)
>
> *The lessons in history of math change my beliefs about math because I realized that math is everywhere."* (EHM05)
>
> *"Sometimes I have this belief that some concepts in math are not applicable to the real-life situation. As we had discussed the history behind sine and cosine, ma'am, has also mentioned a swing. Then I understand maximum and minimum. I can definitely say that the lessons are applicable to real-life situations.* (EHF03)
>
> *"I believe that math is a difficult subject. Our lessons in history of math change my views and helped me a lot."* (EHM06)

A student in the high ability group expressed that his beliefs in mathematics was not changed by the integration of history of mathematics in their Algebra and Trigonometry, rather it had encouraged him to try again.

> *"I believe that math challenges me. Our history lessons in math did not change my beliefs but has given me more reasons to be challenged with math. Sometimes I fail but I still try and try until I get the correct answer.* (EHF08)

Interrelationships Among the Dependent Variables. Six variables were considered in this study. For purposes of analysis, these variables are coded as follows:

V_1 = Mathematics Achievement
V_2 = Attitude towards Mathematics
V_3 = Perceptions on the Usefulness of Mathematics
V_4 = Anxiety in Mathematics
V_5 = Beliefs toward Mathematics
V_6 = Confidence in Learning Mathematics

Table 20 shows the correlation coefficients signifying the degree of relationship that exists between pair of variables. The analysis in this section was based on the posttest scores of the 118 students.

Achievement in mathematics, attitudes toward mathematics, perception of the usefulness of mathematics, confidence in learning mathematics, beliefs about mathematics and anxiety in mathematics is positively and significantly interrelated. Generally, a student who has a high achievement in mathematics has a positive attitude towards mathematics, is less anxious, perceived mathematics as useful in real life situation has improved beliefs toward mathematics and has confidence in learning mathematics.

Table 20. Intercorrelation Matrix (Dependent Variables)

	V_1	V_2	V_3	V_4	V_5	V_6
V_1	1					
V_2	.358*	1				
V_3	.432*	.505*	1			
V_4	.242*	.308*	.293*	1		
V_5	.149**	.544*	.462*	-0,003	1	
V_6	.132**	.367*	.356*	.536*	.113**	1

* Significant at 0.005
** Significant at 0.01

Non-cognitive variables such as attitude towards mathematics, perceptions of the usefulness of mathematics, anxiety in mathematics, beliefs about mathematics and confidence in learning mathematics are positively and significantly related to one another. Students, who have positive attitude towards mathematics, perceive mathematics as useful, are less anxious, have high confidence in learning mathematics, and have better beliefs toward mathematics. On the whole, the dependent variables are significantly interrelated.

Conclusions

The following conclusions are drawn based on the findings of the study:

1. Integrating history of mathematics in the instructional materials and in the teaching of Algebra and Trigonometry did not have a significant effect on students' achievement in mathematics.

2. The achievement of those taught Algebra and Trigonometry with history differed significantly for students with high and low ability in favor of the high ability students. The achievement of male and female students did not differ significantly.
3. The students in the high ability group taught with history of mathematics perceived mathematics to be significantly useful in real-life situations. Their attitude towards mathematics, confidence in learning mathematics, mathematics anxiety and beliefs about mathematics did not significantly differ from those students who were taught Algebra and Trigonometry without history of mathematics on the same ability level.
4. The high and low ability students differed significantly in their mean attitude towards mathematics, confidence to learn mathematics, perception of the usefulness of mathematics, mathematics anxiety and beliefs toward mathematics scores.
5. The male and female students across treatment from different ability levels showed no significant difference in their affective learnings such as attitude towards mathematics, perception of the usefulness of mathematics, confidence in learning mathematics, mathematics anxiety and beliefs about mathematics.

6. There is a significant interrelationship among the variables – mathematics achievement, attitude towards mathematics, perception of the usefulness of mathematics, confidence in learning mathematics, anxiety in mathematics and beliefs about mathematics.

Recommendations

The findings of this study suggest the following recommendations:

1. This study revealed that historical topics in mathematics did not improve students' achievement in mathematics though the students in the experimental group felt the learning process improved their attitudes toward and views about mathematics, perception on the usefulness of mathematics, confidence in learning mathematics and beliefs about mathematics. This indicates the possibility of a gap between what is taught and what is assessed. The history of mathematics, according to NCTM (1989), allows students to study communicating, connecting and valuing mathematics. It motivates students to learn because learning takes place in people who are motivated (Gagne, 1988). History of mathematics provides concept formation (Chi-kai Lit, 2000), concept change (Wandersee, 1990) and authentic problem solving abilities (Fraser, Williamson and Tobin, 1987). These criteria processes obtained from history of mathematics cannot be fully measured by a standardized achievement test specifically by a multiple choice test which relies on exercises, practice and drills. The achievement test of this study is a standardized multiple choice test of four options. The test failed to solicit students' explanations and solutions on how they arrived at their answers. This is the limitation of the study's achievement test. Because of this limitation, the test was not able to capture what history of mathematics tried to inculcate on the students. The test was not able to capture authentic problem solving abilities and concept formation of students. A non multiple choice test may have brought out a better performance from the students exposed to history of mathematics. NCTM (1989) recommends that open-ended problems be part of the curriculum and assessment. In an open-ended question, the students must generate an example, solution, or explanation for which many

possible answers are correct (Senk, Beckman and Thompson, 1997). In line with this, an achievement test with open-ended questions or problems may be the best fit to be used in assessing what the students had learned from history of mathematics. It is recommended that an achievement test in Algebra and Trigonometry consisting of open-ended problems and questions will be used to assess important process criteria other than the standard criteria which rely heavily on exercises, practice and drills. In this way, the performance of students exposed to history of mathematics may have surfaced better. An achievement test shown here is an improvement of the achievement test used in this study. Notice that the former achievement test had 40 items; however, the improved achievement test contains 10 items only. Test with open-ended problems, according to Thompson, Beckman and Senk (1997), should contain fewer items since open-ended items are more substantive than the usual problem of unique answer and solution.

2. In this study, the control group seems to have been given more time to do drill exercises and practice which was measured by the achievement test, whereas the experimental group seem to have been deprived of the exercises and practice because they were doing history. If this study is replicated or if teachers decided in integrating history in their mathematics classes, it is recommended that homework on drill exercises and practice be given to the experimental group to offset the extra advantage of exercises enjoyed by the control group. This is to balance the extra time spent by the experimental group on history of mathematics.
3. The conventional approach to the measurement of affective variables using standardized scales or inventory is inadequate. The result of the quantitative measurement did not agree with the qualititative data on students' feelings about mathematics. A deliberate strategy to the measurement of affective behaviors should be adopted. A study towards the development of this type of approach in measurement is recommended to future researchers.
4. Sixteen hours of teaching mathematics with history may not be enough for students to get used to the new method. This study recommends that the amount of time for teaching mathematics with history be increased so that more historical issues and problems may be given to students.

5. This study showed the importance of linking affective and cognitive aspects in mathematics education. American and Chinese educators have paid considerable attention to this issue. It is high time for Filipino psychologists and mathematics educators to work together so that effective and practical approaches on this respect be developed for the improvement of mathematics education in the Philippines.
6. The teacher plays an important role in the teaching of history of mathematics (Jones, 1969). If this study is replicated, it is recommended that teachers' beliefs about history of mathematics (HOM) and its teaching be part of the design of the study. The result may shed light on teachers' beliefs about HOM, the effect of this belief to his teaching HOM and to the attitude, confidence and beliefs of the students.
7. Considering the encouraging results from the qualitative analysis of data, it is recommended that a follow-up study employing interpretive methods using participant observational fieldwork in order to identify specific causal linkages that were not identified by experimental methods and develop new theories about causes and other influences in the patterns that were identified in the survey data.
8. Likewise, it is recommended to future researchers that history in mathematics instructional materials be developed in geometry, statistics and basic mathematics and a similar study on these areas be conducted.
9. This study had not shown the effect of teaching history of mathematics on students' communication skills. It is recommended that future research along this line be made.
10. How successful the introduction of history of mathematics is depends largely on how it is introduced (Jones, 1969). History of mathematics is not a panacea (Chi-Kai Lit et. al, 2000). No single method can solve all the problems in classroom mathematics learning and each of these can only address a part of them. It is recommended that by identifying the strength and limitation of this teaching strategy (integration of history of mathematics), a teacher can employ it to complement and supplement other methods of teaching mathematics. "The ideal teacher knows well his/her particular discipline, but is ready to transcend it." (BEC, 2002 p.9). In this case, the teacher should become a thinker, an evaluator as well as a curriculum designer. (Wong and Su, 1995; cited by Chi-kai Lit, 2000, p. 47). The development of

curriculum concepts through history should be of great help in designing the teaching of mathematics. As Swetz put it: "*Learning is both cognitive and affective. So, too, are the mathematics problems of History*" (1989, p. 376).

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