TMJM THE MINDANAWAN Journal of Mathematics

The Mindanawan Journal of Mathematics
Official Journal of the Department of Mathematics and Statistics
Mindanao State University-Iligan Institute of Technology
ISSN: 2094-7380 (Print) | 2783-0136 (Online)

Vol. 4 (2022), no. 2, pp. 69–80

A Nonparametric Test for Detecting Univariate Self-Exciting Threshold Autoregressive (SETAR)-Type Nonlinearity in Time Series

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Received: 13th December 2022 Revised: 19th January 2023

Abstract

A number of parametric tests for detecting SETAR (self-exciting threshold autoregressive) - type nonlinearity have been developed in the literature including those of Keenan (1985), Petrucelli and Davies (1986), Tsay (1986, 1989) and Luukkonen et al. (1988). These tests are test-based approaches which require distribution of the particular parametric test. In this paper, a nonparametric test procedure for testing SETAR-type nonlinearity is proposed. The nonparametric test procedure is based on the concept of a model selection criterion, the Akaike's information criterion (AIC), in which the problem of detecting the presence of threshold effects is viewed as a model selection problem among two competing models given by the linear specification and its threshold counterpart. The performance of the proposed test is evaluated by means of simulations. The merits, in terms of size and power, of the proposed test are evaluated relative to Keenan's test and Tsay's F test. The simulation results indicate that the proposed nonparametric test has comparable power to the parametric tests when the entire data generating process is securely stationary and the sample sizes are sufficiently large.

1 Introduction

Since the earlier contribution of Box and Jenkins (1970), stationary time series models like the Autoregressive Moving Average (ARMA) models have been the central focus across diverse fields of endeavor, for which it becomes the standard statistical tools for various time series analysis. However, it is widely recognized that the class of ARIMA models may fail to capture fully the dynamics of real phenomena since these data are often characterized by strong nonlinear components (Corduas, 1994). Tong (2010) studied real-world time series like the Iceland Jokulsa River System data and Mackenzie River lynx data which exhibit some characteristics that are not shown by linear processes such as time irreversibility, non-normality, asymmetric cycles, nonlinear relationship between lagged variables, and sensitivity to initial conditions. With these shortcomings highlighted above, recent developments on nonlinear time series models

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2020 Mathematics Subject Classification: 62G05, 62M10
Keywords and Phrases: SETAR, nonparametric, nonlinearity, time series

-This research is supported by the DOST-ASTHRDP Grant

and methodology arise such that these nonlinear characteristics have been successfully modeled (Tong, 1990).

This paper focuses on one useful class of nonlinear models introduced by Howell Tong (1978), the Self-Exciting Threshold Autoregressive (SETAR) model, a special case of Threshold Autoregressive (TAR) model. It is specified by the following equation:

$$Y_t = \phi_0^{(j)} + \sum_{i=1}^{p} \phi_i^{(j)} Y_{t-i} + \varepsilon_t^{(j)}, \qquad \text{if } r_{j-1} \leq Y_{t-d} \leq r_j, \tag{1}$$

where $j = 1, 2, ..., k,\ t = p+1, ..., n,\ d \leq p$ are both positive integers, $\varepsilon_t \sim$ i.i.d $N(0, \sigma^2_{\varepsilon_t})$. The parameters p and d denote the autoregresive (AR) order and the delay parameter, respectively. Furthermore, k is the number of regimes, $r_j s$ are the threshold paramaters such that $-\infty \leq r_0 < r_1 < ... < r_k = +\infty$, and n is the number of observations. In this way, different regimes possess different AR(p) models. If $\phi_i^{(j)} = \phi_i^{(s)}$, for each $i = 0, 1, ..., p$ and $j \neq s = 1, 2, ..., k$, then the model reduces to a linear AR(p) process. A more generalized version of the SETAR model allows different order of AR models inside different regimes. With threshold parameters, $r_j s$, the process switches among different linear autoregressive models.

The SETAR model is a class of piecewise linear autoregressive model that can effectively capture jump phenomena, amplitude-dependent frequency, and limit cycles. It is piecewsie linear, not in time, but in the space of the threshold variable. In application, there has been growing interests in exploiting potential forecast gains from the nonlinear structure of SETAR models (Hung et al., 2009).

With the emerging interest on nonlinear time series models like the SETAR model and its inherently flexible properties, there seems to have high probability of getting a spuriously good fit to any time series data. Hence, it is very important to conduct preliminary evaluation of the model adequacy whether or not a univariate or multivariate time series may be generated by a linear model against the alternative that they were nonlinearly related instead before building a complex nonlinear model.

A number of parametric tests have already been developed in the literature exhibiting SETAR-type nonlinearity in observed time series including those of Keenan (1985), Tsay (1986), Petrucelli and Davies (1990), Luukkonen et al. (1988). Basically, these parametric tests are based solely on strong distributional assumptions and analytical formulas. In practical statistical situations, traditional parametric approaches to inference is sometimes less ideal in cases where distributional assumptions are violated. With these reasons, developing a nonparametric test statistics is indeed indispensable. Due to the reliance on fewer assumptions and the model structure of the data is not specified a *priori*, nonparametric statistics are more robust and simpler, and its applicability is much wider than the parametric test statistics.

On the basis of the aforementioned premises, a nonparametric test to detect SETAR-type nonlinearity based on the Akaike Information Criterion (AIC) is proposed. This is motivated from the fact that AIC is a model selection criterion. Thus, this proposed test takes a model-selection-based approach in which the problem of detecting the presence of threshold effects is viewed as a model selection problem among two competing models given by the linear specification and its threshold counterpart.

2 The Proposed Methodology

The proposed nonparametric test is model specific. For a stationary time series $\{X_t\}$, we test the following hypotheses:

$$H_0 : \text{AR}(p) \text{ model fits the time series data.}$$
$$H_1 : \text{SETAR}(2; p, d) \text{ model fits the time series data.}$$

Now, suppose a SETAR$(2; p, d)$ model fits the time series data $\{X_t\}$ with

$$\begin{aligned} X_t =& (\phi_0^{(1)} + \sum_{i=1}^{p} \phi_i^{(1)} X_{t-i}) I(X_{t-d} \leq r) \\ &+ (\phi_0^{(2)} + \sum_{i=1}^{p} \phi_i^{(2)} X_{t-i}) I(X_{t-d} > r) + \varepsilon_t \end{aligned} \tag{2}$$

where $t = p+1, ..., n$, and $\varepsilon_t \overset{iid}{\sim} N(0, 1)$.
Under H_0, the model in (2) reduces to linear AR(p) model as

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \varepsilon_t. \tag{3}$$

From model (3), the conditional log-likelihood function of ϕ and σ_ε^2 is given by

$$\ell(\phi, \sigma_\phi^2) = -\frac{n}{2} \ln(\sigma_\varepsilon^2) - \frac{1}{2\sigma_\varepsilon^2} S(\phi) - \frac{n}{2} \ln(2\pi).$$

Maximizing the function with respect ϕ and σ_ε^2 and substituting it to the conditional log-likelihood function gives us

$$\ell(\hat{\phi}, \widehat{\sigma_\varepsilon^2(AR)}) = -\frac{n}{2} \ln \widehat{\sigma_\varepsilon^2(AR)} - \frac{n}{2}(1 + \ln 2\pi).$$

Hence, the computed AIC of the AR model is given as

$$AIC_{AR} = n \ln \widehat{\sigma_\varepsilon^2(AR)} + 2(p+1) + n\left(1 + \ln 2\pi\right),$$

where $\hat{\sigma}_\varepsilon^2$ is the maximum likelihood estimate of the residuals, and $2(p+1)$ is the number of parameters in the AR model. Using similar argument above, under H_1, the computed AIC of the SETAR model is

$$AIC_{SETAR} = n \ln \widehat{\sigma_\varepsilon^2} + 2(2p+1) + n\left(1 + \ln 2\pi\right).$$

Note that if the fitted model is correctly selected, the computed AIC is minimal. Thus, H_0 is not rejected when $AIC_{AR} \leq AIC_{SETAR}$. Otherwise, reject H_0. The algorithm of the proposed nonparametric test is given as follows:

1. Estimate the parameters of the SETAR $(2; p, d)$ model in (2) and predict X_t by

$$\begin{aligned} \hat{X}_t =& (\hat{\phi}_0^{(1)} + \sum_{i=1}^{p} \hat{\phi}_i^{(1)} X_{t-i}) I(X_{t-d} \leq r) \\ &+ (\hat{\phi}_0^{(2)} + \sum_{i=1}^{p} \hat{\phi}_i^{(2)} X_{t-i}) I(X_{t-d} > r) \end{aligned} \tag{4}$$

   where $t = p+1, ..., n$, and $\hat{\Phi}$ is the estimate of the corresponding parameters. Let $AIC_{DIFF} = AIC_{AR} - AIC_{SETAR}$ be the parameter of interest. Compute AIC_{DIFF} from the predicted $\hat{X}_t$. Construct centered residual ε_t from model (4).

2. Generate R bootstrap samples from the centered residual ε_t, for $t = p+1, ..., n$, to obtain simulated innovation ε_t^*.

3. Generate R series for each bootstrap sample in Step 2 using the estimated model (4). The series $X_t^* = (X_{p+1}^*, ..., X_n^*)$ will be simulated as follows:

   (i) Initialize $X_0^*, X_1^*, ..., X_{t-p}^*$.

   (ii) Then

$$\begin{aligned}\hat{X}_t^* =& (\hat{\phi}_0^{(1)} + \sum_{i=1}^{p} \hat{\phi}_i^{(1)} X_{t-i}^*) I(X_{t-d}^* \leq r) \\ &+ \ (\hat{\phi}_0^{(2)} + \sum_{i=1}^{p} \hat{\phi}_i^{(2)} X_{t-i}^*) I(X_{t-d}^* > r) + \varepsilon_t^*.\end{aligned} \tag{5}$$

4. Estimate the parameters of the SETAR $(2; p, d)$ model for every simulated time series in Step 3 and compute

$$\widehat{AIC}^*_{DIFF} = \widehat{AIC}^*_{AR} - \widehat{AIC}^*_{SETAR}.$$

   for the time series $\{X_t^*\}$ in (5).

5. Sort R bootstrap parameter estimates $\widehat{AIC}^*_{DIFF}$ from Step 4 in either ascending or descending order and find the appropriate percentile for the upper limit of the distribution of $\widehat{AIC}^*_{DIFF}$ to construct the corresponding interval.

6. Reject the null hypothesis that the AR(p) model fits the time series data with $(1-\alpha)*100\%$ coverage probability if more than ($\alpha\%$) of the intervals fail to contain the computed $\widehat{AIC}_{DIFF}$ in Step 1.

3 Results and Discussion

3.1 Simulation Studies

Simulation experiments were conducted to evaluate the performance of the proposed nonparametric test procedures based on the concept of the Akaike's information criterion (AIC) for SETAR-type nonlinearity in terms of their size and power of the test relative to the parametric tests introduced in this paper. The sample sizes are set to $T = 50, 100, 150, 300$, and 500, and the number of bootstrapped replicates is $R = 200$. As pointed out by Efron and Tibshirani (1981), as the number of bootstrapped samples $m \to \infty$, the bootstrapped estimate of the sampling distribution of the statistic approaches the sampling distribution of the original estimator, but little improvement in the approximation occurs when m exceeds 50 to 200 samples. For each realization of sample size T in the simulation, $50 + T$, $100 + T$, $150 + T$, $300 + T$, and $500 + T$ observations for $T = 50, 100, 150, 300$ and $T = 500$, respectively, are generated and the first $50, 100, 150, 300$ and 500 observations, respectively, are discarded to avoid dependence on the initial value, that is, setting X_t and ε_t equal to zero for $t \leq 0$. The test will be performed with $p = d = 1$, $r = 0$, no intercept term (that is, $\phi_0^{(1)} = \phi_0^{(2)} = 0$) in the data generated by the model in (2).

In the simulation process, comparison between the existing parametric tests (Keenan's test and Tsay-F test) and the proposed nonparametric test are made based on the size and power of the test.

3.2 Size of the Tests

To compare the performance in terms of size of the tests, a SETAR$(2;1;1)$ model without marked volatility given by

$$X_t = \begin{cases} \phi_1^{(1)} X_{t-1} + \varepsilon_t & \text{if } X_{t-1} \leq 0, \\ \phi_1^{(2)} X_{t-1} + \varepsilon_t & \text{if } X_{t-1} > 0, \end{cases}$$

is used to generate the data with $\phi_1^{(1)} = \phi_1^{(2)} = 0.5$ (securely stationary process in each regime) and $\phi_1^{(1)} = \phi_1^{(2)} = 0.1$ (nearly independent process in each regime) , and $\varepsilon_t \overset{iid}{\sim} N(0,1)$. This SETAR model comprises AR processes with the same parameter values in each regime, that is, $\phi_1^{(1)} = \phi_1^{(2)}$. Thus, this indicates that the data generating process is non-SETAR or AR to be precise. The size of the test is the probability of Type 1 error (the probability of rejecting H_0 when it is true). Hence, the empirical frequencies of rejecting the null hypothesis should be low for this model.

3.2.1 Securely Stationary Process

Table 1 shows the empirical frequencies of rejecting the null hypothesis of an AR process based on $N = 100$ replications with 1% and 5% critical values, and sample sizes of $T = 50, 100, 150, 300$, and 500.

Table 1: Empirical Frequencies of Rejecting an AR Model based on $N = 100$ replications of a SETAR(2;1;1) Model with $\phi_1^{(1)} = \phi_1^{(2)} = 0.5$

			$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	T	K	F	NP	K	F	NP
0.5	0.5	50	18	1	0	6	0	0
0.5	0.5	100	23	5	1	9	0	0
0.5	0.5	150	21	6	2	7	3	0
0.5	0.5	300	22	8	15	11	2	1
0.5	0.5	500	20	11	20	10	4	1

T: Sample Size, K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

As depicted in Table 1, we observe that when the data generating process considers $\phi_1^{(1)} = \phi_1^{(2)} = 0.5$ (securely stationary process in each regime), the empirical frequencies of the nonparametric test (NP) in rejecting the null hypothesis were remarkably lower than those of Keenan and Tsay-F test for samples of size $T = 50, 100, 150, 300$ and $T = 500$. This indicates that the proposed test has smaller Type 1 errors when the data generating process is of an AR process.

3.2.2 Nearly independent process

Table 2 shows the empirical frequencies of rejecting the null hypothesis of an AR process based on $N = 100$ replications with 1% and 5% critical values, and sample sizes of $T = 50, 100, 150, 300$, and 500.

Based on the results obtained in Table 2, we observe that compared to the nonparametric test, the parametric tests (Keenan, Tsay-F) performed better in identifying the AR models when the data generating process considers $\phi_1^{(1)} = \phi_1^{(2)} = 0.1$ (nearly independent process in each regime) as the sample size T increases, since the empirical frequencies of the parametric

Table 2: Empirical Frequencies of Rejecting an AR(1) Model based on $N = 100$ replications of a SETAR(2;1;1) Model with $\phi_1^{(1)} = \phi_1^{(2)} = 0.1$; 5% and 1% critical values

			$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	T	K	F	NP	K	F	NP
0.1	0.1	50	4	8	13	4	5	5
0.1	0.1	100	9	9	30	5	6	14
0.1	0.1	150	6	9	43	2	4	15
0.1	0.1	300	9	5	72	3	0	49
0.1	0.1	500	10	3	94	1	1	82

T: Sample Size, K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

tests in rejecting the null hypothesis were remarkably lower than of the nonparametric test, when there is no change in the parameter values of the upper regime ($\phi_1^{(2)}$) relative to that of the lower regime ($\phi_1^{(1)}$). This indicates that the parametric tests have smaller Type 1 errors than of the proposed nonparametric test when the data generating process is of an AR process.

3.3 Power of the Tests

To compare the performance in terms of the power of the tests, a SETAR $(2; 1; 1)$ model without marked volatility given by

$$X_t = \begin{cases} \phi_1^{(1)} X_{t-1} + \varepsilon_t & \text{if } X_{t-1} \leq 0, \\ \phi_1^{(2)} X_{t-1} + \varepsilon_t & \text{if } X_{t-1} > 0, \end{cases}$$

is used to generate the data, where $\varepsilon_t \overset{iid}{\sim} N(0, 1)$. Hence, the empirical frequencies of rejecting the null hypothesis should be high for this model. Moreover, a critical important aspect in evaluating the performance of the tests in terms of their power is the change of the parameter values for $\phi_1^{(1)}$ as for the parameter value for $\phi_1^{(2)}$ in order for the existing parametric tests (Keenan, Tsay-F) and the proposed nonparametric test to detect a SETAR-type nonlinearity on a given time series. Hence, different parameter values for the upper regime ($\phi_1^{(2)}$) relative to that of the lower regime $\phi_1^{(1)}$ are set for this study.

3.3.1 Securely Stationary Process

Tables 3 to 7 show the empirical frequencies of rejecting an AR(1) model based on $N = 100$ replications of a SETAR$(2; 1; 1)$ Model, with parameter combinations for $\phi_1^{(1)}$ and $\phi_1^{(2)}$: $\phi_1^{(1)} = 0.5$ (securely stationary), $\phi_1^{(2)} = 0, -0.5, -1, -2$.

Table 3: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.5$ (securely stationary); $T = 50$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.5	-2.0	26	26	49	16	18	26
0.5	-1.0	29	19 4	25	19	10	5
0.5	-0.5	18	12	11	7	5	4
0.5	0	3	6	5	0	1	0

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 4: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.5$ (securely stationary); $T = 100$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.5	-2.0	33	40	74	16	18	26
0.5	-1.0	52	42	47	47	27	27
0.5	-0.5	53	28	23	31	9	8
0.5	0	6	1	14	2	1	5

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 5: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.5$ (securely stationary); $T = 150$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.5	-2.0	19	43	77	16	37	60
0.5	-1.0	59	53	69	52	38	42
0.5	-0.5	66	33	53	41	15	30
0.5	0	5	5	26	1	1	11

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 6: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.5$ (securely stationary); $T = 300$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.5	-2.0	35	46	93	30	42	88
0.5	-1.0	89	92	94	87	89	83
0.5	-0.5	98	65	79	93	43	59
0.5	0	1	3	43	1	0	23

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Based on the results depicted from Table 3 to 7, we observe that the nonparametric test performed better than other tests since it has higher emprical frequencies in rejecting the null hypothesis than those of Keenan and Tsay-F test, when the parameter value of the upper regime ($\phi_1^{(2)}$) relative to that of the lower regime ($\phi_1^{(1)}$) decreases significantly. In addition, the power

Table 7: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.5$ (securely stationary); $T = 500$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.5	−2.0	34	44	100	32	41	100
0.5	−1.0	99	99	100	97	98	100
0.5	−0.5	100	81	95	99	58	84
0.5	0	6	11	64	1	1	43

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

of the nonparametric test increases as the sample size also increases, which is evidently shown in the table.

3.3.2 Nearly Independent Process

Tables 8 to 12 show the empirical frequencies of rejecting an $AR(1)$ model based on $N = 100$ replications of a SETAR(2; 1; 1) Model, with parameter combinations for $\phi_1^{(1)}$ and $\phi_1^{(2)}$: $\phi_1^{(1)} = 0.1$ (nearly independent), $\phi_1^{(2)} = 0, -0.5, -1, -2$.

Table 8: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.1$ (nearly independent); $T = 50$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.1	−2.0	88	86	1	82	71	0
0.1	−1.0	60	42	0	42	20	0
0.1	−0.5	15	18	4	7	13	1
0.1	0	3	12	10	2	7	5

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 9: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.1$ (nearly independent); $T = 100$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.1	−2.0	92	98	1	91	94	1
0.1	−1.0	90	73	1	83	50	0
0.1	−0.5	29	24	2	17	12	1
0.1	0	7	11	25	3	4	9

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

From the results obtained in Table 8 to 12, when the data generating process considers $\phi_1^{(1)} = 0.1$ (nearly independent), the empirical frequencies of the nonparametric test in rejecting the null hypothesis were remarkably low even for large samples of size $T = 500$ as the parameter value of the upper regime relative to that of the lower regime decreases significantly. The low frequency of the proposed test may be due to the fact that the data generating process is nearly independent process.

Table 10: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.1$ (nearly independent); $T = 150$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.1	-2.0	97	97	1	97	97	1
0.1	-1.0	98	85	1	94	72	0
0.1	-0.5	34	20	3	15	10	0
0.1	0	4	10	34	2	6	17

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 11: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.1$ (nearly independent); $T = 300$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.1	-2.0	99	100	0	99	100	0
0.1	-1.0	98	96	0	98	96	0
0.1	-0.5	73	51	3	62	31	2
0.1	0	4	8	67	0	3	39

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 12: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.1$ (nearly independent); $T = 500$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.1	-2.0	100	100	0	100	100	0
0.1	-1.0	99	99	0	99	98	0
0.1	-0.5	86	68	9	81	49	1
0.1	0	2	5	94	2	1	72

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

3.3.3 Nearly Nonstationary Process

Tables 13 to 17 show the empirical frequencies of rejecting an AR(1) model based on $N = 100$ replications of a SETAR(2; 1; 1) Model, with parameter combinations for $\phi_1^{(1)}$ and $\phi_1^{(2)}$: $\phi_1^{(1)} = 0.9$ (nearly inonstationary), $\phi_1^{(2)} = 0, -0.5, -1, -2$.

Table 13: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.9$ (nearly nonstationary); $T = 50$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.9	-2.0	27	38	23	18	30	12
0.9	-1.0	30	24	7	14	10	3
0.9	-0.5	13	8	6	3	1	2
0.9	0	2	4	1	0	0	1

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 14: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.9$ (nearly nonstationary); $T = 100$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	FF	NP	K	F	NP
0.9	-2.0	40	78	48	25	64	30
0.9	-1.0	44	48	10	24	23	3
0.9	-0.5	34	28	7	16	5	2
0.9	0	2	1	0	0	0	1

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 15: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.9$ (nearly nonstationary); $T = 150$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.9	-2.0	56	88	62	33	70	35
0.9	-1.0	53	54	14	26	28	10
0.9	-0.5	40	21	6	17	5	2
0.9	0	1 2	0	0	0	0	3

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Table 16: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.9$ (nearly nonstationary); $T = 300$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.9	-2.0	66	98	62	48	92	51
0.9	-1.0	80	91	21	66	71	3
0.9	-0.5	83	58	10	66	31	1
0.9	0	2	2	7	0	0	1

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

Based from the results depicted in Table 13 to 17, when the data generating process considers $\phi_1^{(1)} = 0.9$ (nearly nonstationary), the power of the nonparametric test is not so remarkable as compared to the parametric tests (Keenan and Tsay-F) even for sufficiently large samples of

Table 17: Empirical Frequencies of Rejecting an AR(1) Model on $N = 100$ replications of a SETAR(2; 1; 1) Model with $\phi_1^{(1)} = 0.9$ (nearly nonstationary); $T = 500$

		$\alpha = 0.05$			$\alpha = 0.01$		
$\phi_1^{(1)}$	$\phi_1^{(2)}$	K	F	NP	K	F	NP
0.9	-2.0	83	99	80	65	99	69
0.9	-1.0	87	94	26	75	88	11
0.9	-0.5	93	74	17	75	55	7
0.9	0	2	2	10	0	0	2

K: Keenan's test, F: Tsay-F test, NP: Nonparametric test

size $T = 500$, when the parameter value of the upper regime ($\phi_1^{(2)}$) relative to that of the lower regime ($\phi_1^{(1)}$) decreases significantly.

4 Conclusions and Recommendations

In this paper, not one of the tests considered performs best in detecting SETAR-type nonlinearity based on $N = 100$ replications of a SETAR(2; 1; 1) model. However, there are clear differences apparent in the performance of the parametric tests (Keenan, Tsay-F, and Tsay TAR-F) and the proposed nonparametric test.

1. When the SETAR(2; 1; 1) data generating process considers $\phi_1^{(1)} = \phi_1^{(2)} = 0.5$ (securely stationary process in each regime), the proposed nonparametric test tends to exhibit smaller Type 1 error than the parametric tests (Keenan and Tsay-F) when the data generating process is of an AR process with securely stationary process in each regime.

2. When the SETAR(2; 1; 1) data generating process considers $\phi_1^{(1)} = 0.5$ (securely stationary), the nonparametric test performed better than Keenan and Tsay-F test, when the parameter value of the upper regime ($\phi_1^{(2)}$) relative to that of the lower regime ($\phi_1^{(1)}$) decreases significantly. In addition, the power of the nonparametric test increases as the sample size also increases.

In light of the simulation results obtained, it is recommended to use the nonparametric test in real-world data exhibiting stationary SETAR(2;1,1) series without marked volatility to fully appreciate the significant contribution of the proposed test and modify the nonparametric test procedure with marked volatility in the data generating process and investigate its performance in detecting SETAR-type nonlinearity in time series.

Acknowledgements

This research work is supported by the Department of Science and Technology Science Education Institute (DOST-SEI) and the Premier Research Institute of Science and Mathematics (PRISM) - Applied Mathematics and Statistics Group of MSU-Iligan Institute of Technology.

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