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θ_{sw}-Continuity of Maps in the Product Space and Some Versions of Separation Axioms

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Abstract

In this study, the concept of θ_{sw}-open set is introduced and its relationship to the other well-known concepts such as the classical open, θ-open, and ω_θ-open sets is described. The concepts of θ_{sw}-interior and θ_{sw}-closure of a set is also defined and investigated. Related concepts such as θ_{sw}-open and θ_{sw}-closed functions, θ_{sw}-continuous function, θ_{sw}-connected, and some versions of separation axioms are defined and characterized. Finally, the concept of θ_{sw}-continuous function from an arbitrary topological space into the product space is investigated further.

1 Introduction and Preliminaries

The first attempt to substitute concept in topology with concept possessing either weaker or stronger property was done by N. Levine [24] when he introduced the concepts of semi-open set, semi-closed set, and semi-continuity of a function. Several mathematicians then became interested in introducing other topological concepts which can replace the concept of open set, closed set, and continuity of a function.

In 1968, Velicko [27] introduced the concepts of θ-continuity between topological spaces and subsequently defined the concepts of θ-closure and θ-interior of a subset of topological space. The concept of θ-open set and its related topological concepts had been deeply studied and investigated by numerous authors, see [1, 7, 8, 15, 16, 20, 21, 22, 25, 26].

Let $(X, \mathfrak{T})$ be a topological space and $A \subseteq X$. The *θ-closure* and *θ-interior* of A are, respectively, denoted and defined by

$$Cl_\theta(A) = \{x \in X : Cl(U) \cap A \neq \varnothing \text{ for every open set } U \text{ containing } x\}$$

and

$$Int_\theta(A) = \{x \in X : Cl(U) \subseteq A \text{ for some open set } U \text{ containing } x\},$$

where $Cl(U)$ is the closure of U in X. A subset A of X is *θ-closed* if $Cl_\theta(A) = A$ and *θ-open* if $Int_\theta(A) = A$. Equivalently, A is θ-open if and only if $X \setminus A$ is θ-closed.

In 1971, Hoyle and Gentry [19] introduced the class of *somewhat*-continuous functions and *somewhat*-open functions. The *somewhat*-continuous functions, which are generalization of

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continuity requiring nonempty inverse images of open sets to have nonempty interiors instead of being open, have proved to be very useful in topology. Since then, the concepts of *somewhat*-interior and *somewhat*-closure of a subset of a topological space have been subsequently defined and the concept of *somewhat*-open and *somewhat*-closed sets have been used to characterize *somewhat*-continuity, see [5, 6].

A subset U of a space X is said to be *somewhat*-open if $U = \varnothing$ or if there exists $x \in U$ and an open set V such that $x \in V \subseteq U$. A set is called *somewhat*-closed if its complement is *somewhat*-open. Let A be a subset of a space X. The *somewhat*-closure and *somewhat*-interior of A are, respectively, denoted and defined by $swCl(A) = \cap\{F : F \text{ is } somewhat\text{-closed and } A \subseteq F\}$ and $swInt(A) = \cup\{U \subseteq A : U \text{ is } somewhat\text{-open}\}$.

In 1982, Hdeib [18] introduced the concepts of ω-open and ω-closed sets and ω-closed mappings on a topological space. He showed that ω-closed mappings are strictly weaker than closed mappings and also showed that the Lindelöf property is preserved by counter images of ω-closed mappings with Lindelöf counter image of points. The concepts of ω-open sets and its corresponding topological concepts had been studied in several papers, see [2, 3, 4, 9, 10, 11, 12, 13, 14, 23].

In 2010, Ekici et al. [17] introduced the concepts of ω_θ-open and ω_θ-closed sets on a topological space. They showed that the family of all ω_θ-open sets in a topological space X forms a topology on X. They also introduced the notions of ω_θ-interior and ω_θ-closure of a subset of a topological space.

A point x of a topological space X is called a condensation point of $A \subseteq X$ if for each open set G containing x, $G \cap A$ is uncountable. A subset B of X is ω-closed if it contains all of its condensation points. The complement of B is ω-open. Equivalently, a subset U of X is ω-open (resp., ω_θ-open) if and only if for each $x \in U$, there exists an open set O containing x such that $O \setminus U$ (resp., $O \setminus Int_\theta(U)$) is countable. A subset B of X is ω_θ-closed if its complement $X \setminus B$ is ω_θ-open.

A topological space X is said to be *somewhat*-connected (resp., θ-connected, ω-connected, ω_θ-connected) if X cannot be written as the union of two nonempty disjoint *somewhat*-open (resp., θ-open, ω-open, ω_θ-open) sets.
Otherwise, X is *somewhat*-disconnected (resp., θ-disconnected, ω-disconnected, ω_θ-disconnected).

Let $\mathcal{A}$ be an indexing set and $\{Y_\alpha : \alpha \in \mathcal{A}\}$ be a family of topological spaces. For each $\alpha \in \mathcal{A}$, let $\mathcal{T}_\alpha$ be the topology on Y_α. The Tychonoff topology on $\Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ is the topology generated by a subbase consisting of all sets $p_\alpha^{-1}(U_\alpha)$, where the projection map $p_\alpha : \Pi\{Y_\alpha : \alpha \in \mathcal{A}\} \to Y_\alpha$ is defined by $p_\alpha(\langle y_\beta \rangle) = y_\alpha$, U_α ranges over all members of $\mathcal{T}_\alpha$, and α ranges over all elements of $\mathcal{A}$. Corresponding to $U_\alpha \subseteq Y_\alpha$, denote $p_\alpha^{-1}(U_\alpha)$ by $\langle U_\alpha \rangle$. Similarly, for finitely many indices $\alpha_1, \alpha_2, \ldots, \alpha_n$, and sets $U_{\alpha_1} \subseteq Y_{\alpha_1}$, $U_{\alpha_2} \subseteq Y_{\alpha_2}, \ldots, U_{\alpha_n} \subseteq Y_{\alpha_n}$, the subset

$$\langle U_{\alpha_1} \rangle \cap \langle U_{\alpha_2} \rangle \cap \cdots \cap \langle U_{\alpha_n} \rangle = p_{\alpha_1}^{-1}(U_{\alpha_1}) \cap p_{\alpha_2}^{-1}(U_{\alpha_2}) \cap \cdots \cap p_{\alpha_n}^{-1}(U_{\alpha_n})$$

is denoted by $\langle U_{\alpha_1}, U_{\alpha_2}, \ldots, U_{\alpha_n} \rangle$. We note that for each open set U_α subset of Y_α, $\langle U_\alpha \rangle = p_\alpha^{-1}(U_\alpha) = U_\alpha \times \Pi_{\beta \neq \alpha} Y_\beta$. Hence, a basis for the Tychonoff topology consists of sets of the form $\langle B_{\alpha_1}, B_{\alpha_2}, ..., B_{\alpha_k} \rangle$, where B_{α_i} is open in Y_{α_i} for every $i \in K = \{1, 2, ..., k\}$.

Now, the projection map $p_\alpha : \Pi\{Y_\alpha : \alpha \in \mathcal{A}\} \to Y_\alpha$ is defined by $p_\alpha(\langle y_\beta \rangle) = y_\alpha$ for each $\alpha \in \mathcal{A}$. It is known that every projection map is a continuous open surjection. Also, it is well known that a function f from an arbitrary space X into the Cartesian product Y of the family of spaces $\{Y_\alpha : \alpha \in \mathcal{A}\}$ with the Tychonoff topology is continuous if and only if each coordinate function $p_\alpha \circ f$ is continuous, where p_α is the α-th coordinate projection map.

In this paper, given a topology on X, we define a new type of topology on X which is finer than the topology formed by the collection of θ-open sets, but coarser than the given topology on X.

2 θ_{sw}-Open and θ_{sw}-Closed Functions

In this section, we shall determine the connection of θ_{sw}-open set to the classical open, θ-open, and ω_θ-open sets. We shall also define and characterize the concepts of θ_{sw}-open and θ_{sw}-closed functions.

Definition 2.1. Let X be a topological space. A subset A of X is said to be θ_{sw}-open if for every $x \in A$, there exists an open set U containing x such that $swCl(U) \subseteq A$. A subset F of X is called θ_{sw}-closed if $X \setminus F$ is θ_{sw}-open.

Remark 2.2. The following diagram holds for a subset of a topological space.

$$\begin{array}{ccccccc}
\omega_\theta\text{-open} & \longrightarrow & & \longrightarrow & \omega\text{-open} & & \\
\uparrow & & & & \uparrow & & \\
\theta\text{-open} & \longrightarrow & \theta_{sw}\text{-open} & \longrightarrow & \text{open} & \longrightarrow & \textit{somewhat}\text{-open}
\end{array}$$

We remark that the above diagram is also true for their respective closed sets.

The following examples show that the implications in the above diagram (with respect to θ_{sw}-open set) are not reversible. We note that since ω_θ-open and open are two independent notions [17, Example 5], ω_θ-open does not imply θ_{sw}-open.

Example 2.3. (i) Let $X = \{a, b, c, d, e\}$ with topology $\mathcal{T} = \{\varnothing, X, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{c, d, e\}, \{a, b, c, d\}, \{a, c, d, e\}\}$. Then $\{a, b, c\}$ is θ_{sw}-open but not θ-open.

(ii) Let $\mathbb{R}$ be the real line with topology $\mathcal{T} = \{\varnothing, \mathbb{R}, \mathbb{Q}\}$. Then $\mathbb{Q}$ is open but not θ_{sw}-open.

(iii) Let $\mathbb{R}$ be the real line with topology $\mathcal{T} = \{\varnothing, \mathbb{R}, \mathbb{R} \setminus (1, 2), \{1.5\}, \mathbb{R} \setminus (1, 2) \cup \{1.5\}\}$. Then $\mathbb{R} \setminus (1, 2)$ is θ_{sw}-open but not ω_θ-open.

Before showing that the collection of θ_{sw}-open sets forms a topology, we shall consider first the following remark.

Remark 2.4. Let X be a topological space and $A, B \subseteq X$. Then

(i) $swInt(A)$ is *somewhat*-open and $swInt(A) \subseteq A$;

(ii) $swCl(A)$ is *somewhat*-closed and $A \subseteq swCl(A)$;

(iii) $swInt(A)$ is the largest *somewhat*-open set contained in A;

(iv) If $A \subseteq B$, then $swInt(A) \subseteq swInt(B)$;

(v) $x \in swInt(A)$ if and only if there exists a *somewhat*-open set U containing x such that $U \subseteq A$;

(vi) A is *somewhat*-open if and only if $A = swInt(A)$;

(vii) $swInt(swInt(A)) = swInt(A)$;

(viii) $swInt(A \cap B) \subseteq swInt(A) \cap swInt(B)$;

(ix) $swCl(A)$ is the smallest *somewhat*-closed set containing A;

(x) $A \subseteq B$ implies that $swCl(A) \subseteq swCl(B)$;

(xi) $swCl(swCl(A)) = swCl(A)$;

(xii) $swCl(A) \cup swCl(B) \subseteq swCl(A \cup B)$;

(xiii) $swInt(X \setminus A) = X \setminus swCl(A)$;

(xiv) A is *somewhat*-closed if and only if $A = swCl(A)$; and

(xv) $x \in swCl(A)$ if and only if for every *somewhat*-open set U containing x, $U \cap A \neq \varnothing$.

Theorem 2.5. *Let $\mathcal{T}_{\theta_{sw}}$ be a family of all θ_{sw}-open subsets of topological space X. Then, $\mathcal{T}_{\theta_{sw}}$ forms a topology on X.*

Proof. It is not difficult to see that $\varnothing, X \in \mathcal{T}_{\theta_{sw}}$. Now, let $\{A_i : i \in \mathcal{A}\}$ be a family members of $\mathcal{T}_{\theta_{sw}}$. Let $x \in \bigcup_{i \in \mathcal{A}} A_i$. Then $x \in A_j$ for some $j \in \mathcal{A}$. Since A_j is θ_{sw}-open, there exists an open set U containing x such that $swCl(U) \subseteq A_j \subseteq \bigcup_{i \in \mathcal{A}} A_i$. Hence, $\bigcup_{i \in \mathcal{A}} A_i$ is θ_{sw}-open. Next, let $A_1, A_2 \in \mathcal{T}_{\theta_{sw}}$. Let $x \in A_1 \cap A_2$.Then there exist open sets U_1 and U_2, both containing x, such that $swCl(U_1) \subseteq A_1$ and $swCl(U_2) \subseteq A_2$. Note that $U_1 \cap U_2$ is open containing x. Hence, $swCl(U_1 \cap U_2) \subseteq swCl(U_1) \cap swCl(U_2) \subseteq A_1 \cap A_2$. Therefore, $A_1 \cap A_2$ is θ_{sw}-open. Consequently, $\mathcal{T}_{\theta_{sw}}$ is a topology on X. □

Definition 2.6. Let X be topological space and $A \subseteq X$. The θ_{sw}-interior of A is defined and denoted by $Int_{\theta_{sw}}(A) = \bigcup\{U : U$ is an θ_{sw}-open set and $U \subseteq A\}$. We note that by Theorem 2.5, $Int_{\theta_{sw}}(A)$ is the largest θ_{sw}-open set contained in A. Moreover, $x \in Int_{\theta_{sw}}(A)$ if and only if there exists a θ_{sw}-open set U containing x such that $U \subseteq A$.

Definition 2.7. Let X be topological space and $A \subset X$. The θ_{sw}-closure of A is defined and denoted by $Cl_{\theta_{sw}}(A) = \bigcap\{F : F$ is an θ_{sw}-closed set and $A \subseteq F\}$. We note that by Theorem 2.5, $Cl_{\theta_{sw}}(A)$ is the smallest θ_{sw}-closed set containing A.

Remark 2.8. Let X be a topological space and $A, B \subseteq X$. Then

(i) If $A \subseteq B$, then $Int_{\theta_{sw}}(A) \subseteq Int_{\theta_{sw}}(B)$;

(ii) A is θ_{sw}-open if and only if $A = Int_{\theta_{sw}}(A)$;

(iii) $Int_{\theta_{sw}}(A \cap B) = Int_{\theta_{sw}}(A) \cap Int_{\theta_{sw}}(B)$;

(iv) $x \in Cl_{\theta_{sw}}(A)$ if and only if for every θ_{sw}-open subset U containing x, $U \cap A \neq \varnothing$;

(v) $A \subseteq B$ implies that $Cl_{\theta_{sw}}(A) \subseteq Cl_{\theta_{sw}}(B)$;

(vi) A is θ_{sw}-closed if and only if $Cl_{\theta_{sw}}(A) = A$;

(vii) $Cl_{\theta_{sw}}(Cl_{\theta_{sw}}(A)) = Cl_{\theta_{sw}}(A)$;

(viii) $Cl_{\theta_{sw}}(A) \cup Cl_{\theta_{sw}}(B) = Cl_{\theta_{sw}}(A \cup B)$;

(ix) $Int_{\theta_{sw}}(X \setminus A) = X \setminus Cl_{\theta_{sw}}(A)$;

(x) $Cl_{\theta_{sw}}(X \setminus A) = X \setminus Int_{\theta_{sw}}(A)$;

(xi) A is θ_{sw}-open if and only if for every $x \in A$, there exists a basic open set B containing x such that $swCl(B) \subseteq A$;

(xii) $x \in Int_{\theta_{sw}}(A)$ if and only if there exists an open set O containing x, $swCl(O) \subseteq A$; and

(xiii) $x \in Cl_{\theta_{sw}}(A)$ if and only if for every open set U containing x, $swCl(U) \cap A \neq \varnothing$.

Next, we introduce and characterize the concepts of θ_{sw}-open and θ_{sw}-closed functions.

Definition 2.9. Let X and Y be topological spaces. A function $f : X \to Y$ is θ_{sw}-open (resp., θ_{sw}-closed) if $f(G)$ is θ_{sw}-open (resp., θ_{sw}-closed) for every open (resp., closed) set G in X.

In view of Remark 2.2, we have the following results.

Remark 2.10. Let $f : X \to Y$ be a function of topological spaces.

(i) If f is θ_{sw}-open (resp., θ_{sw}-closed) on X, then f is open (resp., closed) on X.

(ii) If f is θ-open (resp., θ-closed) on X, then f is θ_{sw}-open (resp., θ_{sw}-closed) on X.

Remark 2.11. The converse of Remark 2.10 (i) and (ii) do not necessarily hold.

(i) Consider $\mathbb{R}$ with topologies $\mathcal{T}_1 = \{\varnothing, \mathbb{R}, \mathbb{Q}\}$ and $\mathcal{T}_2 = \{\varnothing, \mathbb{R}, \mathbb{Q}, \mathbb{Q} \setminus \{0\}\}$. Define $f : (\mathbb{R}, \mathcal{T}_1) \to (\mathbb{R}, \mathcal{T}_2)$ by $f(x) = x$ for all $x \in \mathbb{R}$. Obviously, f is open on $(\mathbb{R}, \mathcal{T}_1)$. Next, we will show that there exists an open set in $(\mathbb{R}, \mathcal{T}_1)$ such that its image is not θ_{sw}-open in $(\mathbb{R}, \mathcal{T}_2)$. Note that $\mathbb{Q}^c$ is not somewhat open since for every $x \in \mathbb{Q}^c$ the only open set in $(\mathbb{R}, \mathcal{T}_2)$ containing x is $\mathbb{R}$ and $\mathbb{R} \not\subseteq \mathbb{Q}^c$. Now, if $swCl(\mathbb{Q}) = \mathbb{Q}$, then $\mathbb{Q}$ is somewhat close, a contradiction. Thus, $\mathbb{Q} \subset swCl(\mathbb{Q})$. Note also that $swCl(\mathbb{R}) = \mathbb{R} \not\subseteq \mathbb{Q}$. Hence, $\mathbb{Q}$ is not θ_{sw}-open in $(\mathbb{R}, \mathcal{T}_2)$. This means that $f(\mathbb{Q}) = \mathbb{Q}$ is not θ_{sw}-open in $(\mathbb{R}, \mathcal{T}_2)$. Therefore, f is not θ_{sw}-open on $(\mathbb{R}, \mathcal{T}_1)$. Since f is bijective, f is closed but not θ_{sw}-closed on $(\mathbb{R}, \mathcal{T}_1)$.

(ii) Consider the topological spaces $X = Y = \{a, b, c, d, e\}$ with the corresponding topologies $\mathcal{T}_x = \{\varnothing, X, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, c, d\}\}$ and $\mathcal{T}_y = \mathcal{T}$ in Example 2.3 (i). Let $f : (X, \mathcal{T}_x) \to (Y, \mathcal{T}_y)$ be a function defined by $f(x) = x$ for all $x \in X$. By definition of f and θ_{sw}-open set, f is θ_{sw}-open on $(X, \mathcal{T}_x)$. Next, we will show that there exists an open set in $(X, \mathcal{T}_x)$ such that its image is not θ-open in $(X, \mathcal{T}_y)$. Consider the open set $A = \{a, b, c\}$ in $(X, \mathcal{T}_x)$. Then $f(A) = A$. We claim that A is θ_{sw}-open but not θ-open in $(X, \mathcal{T}_y)$. By Example 2.3 (i), A is θ_{sw}-open but not θ-open. Thus, f is not θ-open on $(X, \mathcal{T}_x)$. Since f is bijective, f is θ_{sw}-closed but not θ-closed on $(X, \mathcal{T}_x)$.

Theorem 2.12. *Let X and Y be topological spaces and $f : X \to Y$ be a function. Then the following statements are equivalent:*

(i) *f is θ_{sw}-open on X.*

(ii) *$f(Int(A)) \subseteq Int_{\theta_{sw}}(f(A))$ for every $A \subseteq X$.*

(iii) *$f(B)$ is θ_{sw}-open for every basic open set B in X.*

(iv) *For each $x \in X$ and every open set U in X containing x, there exists an open set V in Y containing $f(x)$ such that $swCl(V) \subseteq f(U)$.*

Proof. (i)⇒(ii): Let $A \subseteq X$. Note that $f(Int(A)) \subseteq f(A)$ and $f(Int(A))$ is θ_{sw}-open. In view of Definition 2.6, $f(Int(A)) \subseteq Int_{\theta_{sw}}(f(A))$.

(ii)⇒(iii): Let B be a basic open set in X. Then $f(B) = f(Int(B)) \subseteq Int_{\theta_{sw}}(f(B)) \subseteq f(B)$. By Remark 2.8 (ii), $f(B)$ is θ_{sw}-open.

(iii)⇒(iv): Let $x \in X$ and U be an open set containing x. Then there exists a basic open set B containing x such that $B \subseteq U$, which implies $f(x) \in f(B) \subseteq f(U)$. By assumption, there exists an open set V containing $f(x)$ such that $swCl(V) \subseteq f(B) \subseteq f(U)$.

(iv)⇒(i): Let U be an open set in X and let $y \in f(U)$. Then there exists $x \in U$ such that $f(x) = y$. By assumption, there exists an open set V containing y such that $swCl(V) \subseteq f(U)$. Hence, $f(U)$ is θ_{sw}-open in Y. Therefore, f is θ_{sw}-open on X. □

Theorem 2.13. *Let X and Y be topological spaces and $f : X \to Y$ be a function. Then the following are equivalent:*

(i) *f is θ_{sw}-closed on X.*

(ii) *$Cl_{\theta_{sw}}(f(A)) \subseteq f(Cl(A))$, for every $A \subseteq X$.*

Proof. (i)⇒(ii): Let $A \subseteq X$. Note that $f(A) \subseteq f(Cl(A))$ and $f(Cl(A))$ is θ_{sw}-closed. In view of Definition 2.7, $Cl_{\theta_{sw}}(f(A)) \subseteq f(Cl(A))$.

(ii)⇒(i): Let F be closed in X. By assumption, $f(F) \subseteq Cl_{\theta_{sw}}(f(F)) \subseteq f(Cl(F)) = f(F)$. By Remark 2.8 (vii), f is θ_{sw}-closed on X. □

Remark 2.14. Let X and Y be topological spaces and $f : X \to Y$ be a bijective function. Then f is θ_{sw}-open on X if and only if f is θ_{sw}-closed on X.

3 θ_{sw}-Continuity of Functions in the Product Space

In this section, we provide a definition of a θ_{sw}-continuous function and its characterization from an arbitrary topological space into the product space.

Definition 3.1. A function $f : X \to Y$ is θ_{sw}-continuous if $f^{-1}(G)$ is θ_{sw}-open for every open set G of Y.

In view of Remark 2.2, the following remark holds.

Remark 3.2. Let $f : X \to Y$ be a function of topological spaces. If f is θ_{sw}-continuous on X, then f is continuous on X.

Remark 3.3. The converse of Remark 3.2 does not necessarily hold.

To see this, consider $\mathbb{R}$ with topology $\mathcal{T} = \{\varnothing, \mathbb{R}, \mathbb{Q}\}$. Define $f : (\mathbb{R}, \mathcal{T}) \to (\mathbb{R}, \mathcal{T})$ by $f(x) = x$ for all $x \in \mathbb{R}$. Obviously, f is continuous on $\mathbb{R}$. Next, we will show that there exists an open set in $(\mathbb{R}, \mathcal{T})$ such that its inverse image is not θ_{sw}-open in $(\mathbb{R}, \mathcal{T})$. Consider the open set $\mathbb{Q}$ in $(\mathbb{R}, \mathcal{T})$. We will show that it is not θ_{sw}-open in $(\mathbb{R}, \mathcal{T})$. By Example 2.3 (ii), $\mathbb{Q}$ is not θ_{sw}-open in $(\mathbb{R}, \mathcal{T})$. That is, $f^{-1}(\mathbb{Q}) = \mathbb{Q}$ is not θ_{sw}-open in $(\mathbb{R}, \mathcal{T})$. Therefore, f is continuous but not θ_{sw}-continuous on $(\mathbb{R}, \mathcal{T})$.

The proofs of the following results are standard, hence omitted.

Theorem 3.4. *Let X and Y be topological spaces and $f : X \to Y$ be a function. Then the following statements are equivalent:*

(i) *f is θ_{sw}-continuous on X.*

(ii) *$f^{-1}(F)$ is θ_{sw}-closed in X for each closed subset F of Y.*

(iii) *$f^{-1}(B)$ is θ_{sw}-open in X for each (subbasic) basic open set B in Y.*

(iv) *For every $p \in X$ and every open set V of Y containing $f(p)$, there exists a θ_{sw}-open set U of X containing p such that $f(U) \subseteq V$.*

(v) *$f(Cl_{\theta_{sw}}(A)) \subseteq Cl(f(A))$ for each $A \subseteq X$.*

(vi) *$Cl_{\theta_{sw}}(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$ for each $B \subseteq Y$.*

Theorem 3.5. *Let X be any topological space and $\chi_A : X \to \mathcal{D}$ the characteristic function of a subset A of X. Then χ_A is θ_{sw}-continuous if and only if A is both θ_{sw}-open and θ_{sw}-closed.*

In the following results, if $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ is a product space and $A_\alpha \subseteq Y_\alpha$ for each $\alpha \in \mathcal{A}$, we denote $A_{\alpha_1} \times \cdots \times A_{\alpha_n} \times \Pi\{Y_i : \alpha \notin K\}$ by $\langle A_{\alpha_1}, \cdots, A_{\alpha_n}\rangle$, where $K = \{\alpha_1, \cdots, \alpha_n\}$.

If $Y = \Pi\{Y_i : 1 \leq i \leq n\}$ is a finite product, denote $A_{\alpha_1} \times \cdots \times A_{\alpha_n}$ by $\langle A_{\alpha_1}, \cdots, A_{\alpha_n}\rangle$.

Theorem 3.6. *Let $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ be a product space. Let $S = \{\alpha_1, \ldots, \alpha_n\}$ be a finite subset of $\mathcal{A}$ and $\varnothing \neq O_\alpha \subseteq Y_\alpha$ for each $\alpha \in \mathcal{A}$. Then $O = \langle O_{\alpha_1}, \ldots, O_{\alpha_n}\rangle$ is somewhat-open in Y if and only if each O_{α_i} is somewhat-open in Y_{α_i}.*

Theorem 3.7. *Let $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ be a product space and $A_\alpha \subseteq Y_\alpha$ for each $\alpha \in \mathcal{A}$. Then $swCl(\Pi\{A_\alpha : \alpha \in \mathcal{A}\}) \subseteq \Pi\{swCl(A_\alpha) : \alpha \in \mathcal{A}\}$.*

Proof. Let $x = \langle a_\alpha \rangle \notin \Pi\{swCl(A_\alpha) : \alpha \in \mathcal{A}\}$. Then $a_\beta \notin swCl(A_\beta)$ for some $\beta \in \mathcal{A}$. This means that there exists a *somewhat*-open set G_β containing a_β such that $G_\beta \cap A_\beta = \varnothing$. By Theorem 3.6, $\langle G_\beta \rangle$ is *somewhat*-open containing x. Hence, $\langle G_\beta\rangle \cap \Pi\{A_\alpha : \alpha \in \mathcal{A}\} = \varnothing$. Thus, $x \notin swCl(\Pi\{A_\alpha : \alpha \in \mathcal{A}\})$. □

Theorem 3.8. *Let $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ be a product space and $A_\alpha \subseteq Y_\alpha$ for each $\alpha \in \mathcal{A}$. Then $Cl_{\theta_{sw}}(\Pi\{A_\alpha : \alpha \in \mathcal{A}\}) \subseteq \Pi\{Cl_{\theta_{sw}}(A_\alpha) : \alpha \in \mathcal{A}\}$.*

Proof. Let $x = \langle a_\alpha \rangle \in Cl_{\theta_{sw}}(\Pi\{A_\alpha : \alpha \in \mathcal{A}\})$. Then for every open set O containing x, $swCl(O) \cap \Pi\{A_\alpha : \alpha \in \mathcal{A}\} \neq \varnothing$. Suppose that there exists $\beta \in \mathcal{A}$ such that $a_\beta \notin Cl_{\theta_{sw}}(A_\beta)$. Then there exists an open set U_β containing a_β such that $swCl(U_\beta) \cap A_\beta = \varnothing$. By Theorem 3.7, $swCl(\langle U_\beta\rangle) \subseteq \langle swCl(U_\beta)\rangle$. It follows that $x = \langle a_\alpha\rangle \in \langle U_\beta\rangle$ and $swCl(\langle U_\beta\rangle) \cap \Pi\{A_\alpha : \alpha \in \mathcal{A}\} = \varnothing$, a contradiction. Thus, $x \in \Pi\{Cl_{\theta_{sw}}(A_\alpha) : \alpha \in \mathcal{A}\}$. □

Theorem 3.9. *Let $Y = \Pi\{Y_i : 1 \leq i \leq n\}$ be a (finite) product space and $A_i \subseteq Y_i$ for each $i = 1, 2, \ldots, n$. Then $\Pi\{Int_{\theta_{sw}}(A_i) : 1 \leq i \leq n\} \subseteq Int_{\theta_{sw}}(\Pi\{A_i : 1 \leq i \leq n\})$.*

Proof. Let $x = \langle a_1, a_2, \ldots, a_n\rangle \in \Pi\{Int_{\theta_{sw}}(A_i) : 1 \leq i \leq n\}$. Then $a_i \in Int_{\theta_{sw}}(A_i)$ for all $i = 1, 2, \ldots, n$. This means that for all $i = 1, 2, \ldots, n$, there exists an open set O_i containing a_i such that $swCl(O_i) \subseteq A_i$. Let $O = \langle O_1, O_2, \ldots, O_n\rangle$, which is an open set in Y containing x. By Theorem 3.7, $swCl(O) = swCl(\langle O_1, O_2, \ldots, O_n\rangle) \subseteq \langle swCl(O_1), swCl(O_2), \ldots, swCl(O_n)\rangle \subseteq \langle A_1, A_2, \ldots, A_n\rangle$. Hence, $x \in Int_{\theta_{sw}}(\Pi\{A_i : 1 \leq i \leq n\})$. □

Theorem 3.10. *Let X be a topological space and $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ a product space. A function $f : X \to Y$ is θ_{sw}-continuous on X if and only if $p_\alpha \circ f$ is θ_{sw}-continuous on X for every $\alpha \in \mathcal{A}$.*

Proof. Suppose that f is θ_{sw}-continuous on X. Let $\alpha \in \mathcal{A}$ and U_α be an open set in Y_α. Since p_α is continuous, ${p_\alpha}^{-1}(U_\alpha)$ is open in Y. Hence, $f^{-1}({p_\alpha}^{-1}(U_\alpha)) = (p_\alpha \circ f)^{-1}(U_\alpha)$ is θ_{sw}-open set in X. Therefore, $p_\alpha \circ f$ is θ_{sw}-continuous for every $\alpha \in \mathcal{A}$.

Conversely, suppose that each coordinate function $p_\alpha \circ f$ is θ_{sw}-continuous. Let G_α be open in Y_α. Then, $\langle G_\alpha\rangle$ is a subbasic open set in Y and $(p_\alpha \circ f)^{-1}(G_\alpha) = f^{-1}({p_\alpha}^{-1}(G_\alpha)) = f^{-1}(\langle G_\alpha\rangle)$ is θ_{sw}-open in X. Therefore, f is θ_{sw}-continuous. □

Corollary 3.11. *Let X be a topological space, $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ a product space, and $f_\alpha : X \to Y_\alpha$ a function for each $\alpha \in \mathcal{A}$. Let $f : X \to Y$ be the function defined by $f(x) = \langle f_\alpha(x)\rangle$. Then f is θ_{sw}-continuous on X if and only if each f_α is θ_{sw}-continuous on X for each $\alpha \in \mathcal{A}$.*

Theorem 3.12. *Let $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ be a product space, $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\} \subseteq \mathcal{A}$, and $\varnothing \neq O_{\alpha_i} \subseteq Y_{\alpha_i}$ for each $\alpha_i \in S$. If each O_{α_i} is θ_{sw}-open in Y_{α_i}, then $O := \langle O_{\alpha_1}, O_{\alpha_2}, \ldots, O_{\alpha_n}\rangle$ is θ_{sw}-open in Y.*

Proof. Let $x = \langle a_\alpha \rangle \in O$. Then $a_{\alpha_i} \in O_{\alpha_i}$ for every $\alpha_i \in S$. This means that for every $\alpha_i \in S$, there exists an open set U_{α_i} containing a_{α_i} such that $swCl(U_{\alpha_i}) \subseteq O_{\alpha_i}$. Let $U = \langle U_{\alpha_i}, U_{\alpha_2}, \dots, U_{\alpha_n} \rangle$. Then $x \in U$ and by Theorem 3.7,

$$swCl(U) \subseteq \langle swCl(U_{\alpha_1}), swCl(U_{\alpha_2}), \dots, swCl(U_{\alpha_n}) \rangle \subseteq O.$$

Thus, O is θ_{sw}-open in Y. □

Theorem 3.13. *Let $X = \Pi\{X_\alpha : \alpha \in \mathcal{A}\}$ and $Y = \Pi\{Y_\alpha : \alpha \in \mathcal{A}\}$ be product spaces and for each $\alpha \in \mathcal{A}$, let $f_\alpha : X_\alpha \to Y_\alpha$ be a function. If each f_α is θ_{sw}-continuous on X_α, then the function $f : X \to Y$ defined by $f(\langle x_\alpha \rangle) = \langle f_\alpha(x_\alpha) \rangle$ is θ_{sw}-continuous on X.*

Proof. Let $\langle V_\alpha \rangle$ be a subbasic open set in Y. Then $f^{-1}(\langle V_\alpha \rangle) = \langle f_\alpha^{-1}(V_\alpha) \rangle$. Since each f_α is θ_{sw}-continuous, $f_\alpha^{-1}(V_\alpha)$ is θ_{sw}-open in X_α. Let $x = \langle x_\beta \rangle \in f^{-1}(\langle V_\alpha \rangle) = \langle f_\alpha^{-1}(V_\alpha) \rangle$. Then $x_\alpha \in f_\alpha^{-1}(V_\alpha)$. Hence, there exists an open set U_α containing x_α such that $swCl(U_\alpha) \subseteq f_\alpha^{-1}(V_\alpha)$. Note that $\langle U_\alpha \rangle$ is open in X contining x. By Theorem 3.7, $swCl(\langle U_\alpha \rangle) \subseteq \langle swCl(U_\alpha) \rangle \subseteq \langle f_\alpha^{-1}(V_\alpha) \rangle = f^{-1}(\langle V_\alpha \rangle)$. This means that $f^{-1}(\langle V_\alpha \rangle)$ is θ_{sw}-open in X. Thus, f is θ_{sw}-continuous on X. □

4 θ_{sw}-Connectedness and Some Versions of Separation Axioms

In this section, we define and characterize the concepts of θ_{sw}-connected, θ_{sw}-Hausdorff, θ_{sw}-regular, and θ_{sw}-normal spaces.

Definition 4.1. A topological space X is said to be θ_{sw}-connected if it is not the union of two nonempty disjoint θ_{sw}-open sets. Otherwise, X is θ_{sw}-disconnected. A subset B of X is θ_{sw}-connected if it is θ_{sw}-connected as a subspace of X.

Theorem 4.2. *Let X be a topological space. Then the following statements are equivalent:*

(i) *X is θ_{sw}-connected.*

(ii) *The only subsets of X that are both θ_{sw}-open and θ_{sw}-closed are $\varnothing$ and X .*

(iii) *No θ_{sw}-continuous function from X to $\mathcal{D}$ is surjective.*

Theorem 4.3. *A topological space X is connected if and only if it is θ-connected.*

Proof. In view of Remark 2.2, connectedness implies θ-connectedness.

Conversely, assume that X is θ-connected. If X were disconnected, then there exist disjoint nonempty open sets G and H such that $X = G \cup H$. This implies that G and H are also closed sets, hence $Cl(G) = G \subseteq G$ and $Cl(H) = H \subseteq H$. Thus, G and H are θ-open sets. Thus X is θ-disconnected, contrary to our assumption. □

In view of Remark 2.2 and Theorem 4.3, the following result holds.

Theorem 4.4. *A topological space X is θ_{sw}-connected if and only if it is θ-connected.*

Remark 4.5. The following diagram holds for a subset of a topological space.

$$\begin{array}{ccccccc}
\omega_\theta\text{-connected} & & \longleftarrow & & \omega\text{-connected} & & \\
\downarrow & & & & \downarrow & & \\
\theta\text{-connected} & \longleftrightarrow & \theta_{sw}\text{-connected} & \longleftrightarrow & \text{connected} & \longleftarrow & \textit{somewhat}\text{-connected}
\end{array}$$

Definition 4.6. A topological space X is said to be

(i) *θ_{sw}-Hausdorff* if given any pair of distinct points p, q in X there exist disjoint θ_{sw}-open sets U and V such that $p \in U$ and $q \in V$;

(ii) *θ_{sw}-regular* if for each closed set F and each point $x \notin F$, there exist disjoint θ_{sw}-open sets U and V such that $x \in U$ and $F \subseteq V$; and

(iii) *θ_{sw}-normal* if for every pair of disjoint closed sets E and F of X, there exist disjoint θ_{sw}-open sets U and V such that $E \subseteq U$ and $F \subseteq V$.

In view of Remark 2.2, every θ_{sw}-Hausdorff (resp., θ_{sw}-regular, θ_{sw}-normal) space is Hausdorff (resp., regular, normal).

Theorem 4.7. *Let X be a topological space. Then the following are equivalent:*

(i) *X is θ_{sw}-Hausdorff.*

(ii) *Let $x \in X$. For $y \neq x$, there exists a θ_{sw}-open set U containing x such that $y \notin Cl_{\theta_{sw}}(U)$.*

(iii) *For each $x \in X$, $C := \cap\{Cl_{\theta_{sw}}(U) : U$ is an θ_{sw}-open set with $x \in U\} = \{x\}$.*

Proof. (i)⇒(ii): For every disjoint points $x, y \in X$, there exist disjoint θ_{sw}-open sets U and V such that $x \in U$ and $y \in V$. By Remark 2.8 (v), $y \notin Cl_{\theta_{sw}}(U)$.

(ii)⇒(iii): Let $x \in X$. Then $x \in C$. Hence, for every $y \neq x$, there exists a θ_{sw}-open set U containing x such that $y \notin Cl_{\theta_{sw}}(U)$. Thus, $y \notin C$. Since y is arbitrary, $C = \{x\}$.

(iii)⇒(i): Let $x, y \in X$ such that $x \neq y$. By assumption, there exists a θ_{sw}-open set U containing x such that $y \notin Cl_{\theta_{sw}}(U)$. By Remark 2.8 (v), there exists a θ_{sw}-open set V containing y such that $U \cap V = \varnothing$. Hence, X is θ_{sw}-Hausdorff. □

Theorem 4.8. *Let X be a topological space. Then the following are equivalent:*

(i) *X is θ_{sw}-regular.*

(ii) *For each $x \in X$ and open set U with $x \in U$, there exists a θ_{sw}-open set V with $x \in V$ such that $V \subseteq Cl_{\theta_{sw}}(V) \subseteq U$.*

(iii) *For each $x \in X$ and closed set F with $x \notin F$, there exists a θ_{sw}-open set V with $x \notin V$ such that $Cl_{\theta_{sw}}(V) \cap F = \varnothing$.*

Proof. (i)⇒(ii): Let $x \in X$ and U be an open set containing x. Then $X \setminus U$ is closed and $x \notin X \setminus U$. By assumption, there exist disjoint θ_{sw}-open sets V and W such that $x \in V$ and $X \setminus U \subseteq W$. By Remark 2.8 (vi), (vii), $Cl_{\theta_{sw}}(V) \subseteq Cl_{\theta_{sw}}(X \setminus W) = X \setminus W$. Moreover, $Cl_{\theta_{sw}}(V) \cap X \setminus U \subseteq Cl_{\theta_{sw}}(V) \cap W = \varnothing$ so that $Cl_{\theta_{sw}}(V) \subseteq U$. Thus, $V \subseteq Cl_{\theta_{sw}}(V) \subseteq U$.

(ii)⇒(iii): Let $x \in X$ and F be a closed set with $x \notin F$. Then $X \setminus F$ is open containing x. By assumption, there exists a θ_{sw}-open set V containing x such that $V \subseteq Cl_{\theta_{sw}}(V) \subseteq X \setminus F$. This means that $Cl_{\theta_{sw}}(V) \cap F = \varnothing$.

(iii)⇒(i): Let $x \in X$ and F be a closed set with $x \notin F$. By assumption, there exists a θ_{sw}-open set V with $x \in V$ such that $Cl_{\theta_{sw}}(V) \cap F = \varnothing$. Note that $X \setminus Cl_{\theta_{sw}}(V)$ is θ_{sw}-open and $F \subseteq X \setminus Cl_{\theta_{sw}}(V)$. Furthermore, $V \cap X \setminus Cl_{\theta_{sw}}(V) = \varnothing$. Hence, X is θ_{sw}-regular. □

Theorem 4.9. *Let X be a topological space. Then the following are equivalent:*

(i) *X is θ_{sw}-normal.*

(ii) *For each closed set A and each open set U containing A, there exists a θ_{sw}-open set V containing A such that $Cl_{\theta_{sw}}(V) \subseteq U$.*

(iii) *For each pair of disjoint closed sets A and B, there exists θ_{sw}-open set U containing A such that $Cl_{\theta_{sw}}(U) \cap B = \varnothing$.*

Proof. (i)⇒(ii): Let A be closed and U be open containing A. Then A and $X \setminus U$ are disjoint closed sets in X. By assumption, there exist disjoint θ_{sw}-open sets V and W such that $A \subseteq V$ and $X \setminus U \subseteq W$ or $(X \setminus W \subseteq U)$. By Remark 2.8 (vi), (vii), $Cl_{\theta_{sw}}(V) \subseteq Cl_{\theta_{sw}}(X \setminus W) = X \setminus W$ so that $Cl_{\theta_{sw}}(V) \subseteq Cl_{\theta_{sw}}(X \setminus W) = X \setminus W \subseteq U$.

(ii)⇒(iii): Let A and B be two disjoint closed sets. Then, $A \subseteq X \setminus B$ and $X \setminus B$ is open. By assumption, there exists a θ_{sw}-open set U containing A such that $Cl_{\theta_{sw}}(U) \subseteq X \setminus B$. This means that $Cl_{\theta_{sw}}(V) \cap B = \varnothing$.

(iii)⇒(i): Let A and B be disjoint closed sets. By assumption, there exists a θ_{sw}-open set U containing A such that $Cl_{\theta_{sw}}(U) \cap B = \varnothing$. Then $B \subseteq X \setminus Cl_{\theta_{sw}}(U)$. Since U and $X \setminus Cl_{\theta_{sw}}(U)$ are disjoint θ_{sw}-opens, X is θ_{sw}-normal. □

A topological space X is said to be a T_1-space if for each $p, q \in X$ with $p \neq q$, there exist open sets U and V such that $p \in U$ and $q \notin U$, and $q \in V$ and $p \notin V$.

Theorem 4.10. *Let X be a T_1-space. Then*

(i) *If X is θ_{sw}-regular, then X is θ_{sw}-Hausdorff; and*

(ii) *If X is θ_{sw}-normal, then X is θ_{sw}-regular.*

Proof. (i): Suppose that X is θ_{sw}-regular. For each $x, y \in X$ with $x \neq y$, there exist disjoint open sets U and V such that $x \in U$ and $y \notin U$, and $y \in V$ and $x \notin V$. This implies that $x \notin X \setminus U$ and $y \notin X \setminus V$. Note that $X \setminus U$ is closed. Since X is θ_{sw}-regular, there exists disjoint θ_{sw}-open sets A and B such that $x \in A$ and $X \setminus U \subseteq B$. Note that $y \in X \setminus U$. Hence, $y \in B$. Thus, X is θ_{sw}-Hausdorff.

We can prove (ii) by following the same argument used in (i). □

5 Conclusion and Recommendations

The paper has introduced the concept of θ_{sw}-open set and described its connection to the other well-known concepts such as the classical open, θ-open, and ω_θ-open sets. The paper has also defined and characterized the concepts of θ_{sw}-open and θ_{sw}-closed functions, θ_{sw}-continuous function, and θ_{sw}-connected, θ_{sw}-Hausdorff, θ_{sw}-regular, and θ_{sw}-normal spaces. Moreover, the paper has formulated a necessary and sufficient condition for θ_{sw}-continuity of a function from an arbitrary space into the product space. A worthwhile direction for further investigation is to establish versions of Urysohn's Lemma and Tietze Extension Theorem for θ_{sw}-open sets. One may also try to investigate θ_{sw}-open and θ_{sw}-closed sets, and other related concepts in a generalized topological space.

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