TMJM THE MINDANAWAN JOURNAL OF MATHEMATICS
OFFICIAL JOURNAL OF THE MINDANAO STATE UNIVERSITY - ILIGAN INSTITUTE OF TECHNOLOGY

Vol. 6 (2024), no. 2, pp. 135–153
https://doi.org/10.62071/tmjm.v6i2.718.
ISSN: 2094-7380 (Print) | 2783-0136 (Online)

Analysis of an Epidemic Model Incorporating Anxiety Dynamics

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Received: 31st December 2024 Revised: 9th January 2025

Abstract

This study formulates and analyzes a COVID-19 disease contagion model, incorporating a psychological variable over a population considering that infected cases can be confirmed or unreported. Using a system of ordinary differential equations, the model describes the impact of anxiety on the disease progression where the contact parameter is defined as a function of anxiety level. We derive the basic reproduction number $\mathcal{R}_0$ and numerical simulations validate our theoretical results. Our findings provide a qualitative understanding of the interplay between psychological states and epidemiological outcomes, offering a novel framework for future research and potential policymaking applications in epidemic response.

1 Introduction

Anxiety is a complex and widely studied psychological phenomenon that affects human behavior in various contexts. Defined as an unpleasant emotional state characterized by feelings of tension, apprehension, and worry, anxiety plays a significant role in social interactions [4]. While moderate levels of anxiety may help individuals avoid potential threats and navigate social situations [9], excessive or inappropriate anxiety can have detrimental effects on social life and overall well-being [15]. Research has highlighted how varying levels of anxiety influence social decision-making, with individuals differing in anxiety levels demonstrating distinct behavioral patterns and autonomic responses during decision-making processes [20]. The impact of anxiety on mental health has become particularly evident in recent years, as the World Health Organization (WHO) reported a 25% global increase in anxiety and depression during the first year of the COVID-19 pandemic [19]. In hindsight, this surge in mental health concerns prompted 90% of countries to incorporate mental health and psychosocial support into their COVID-19 response plans, though substantial gaps remain.

A study assessing the prevalence of anxiety and depression among disadvantaged public university students in New York City revealed alarming levels of psychological distress among those

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2020 Mathematics Subject Classification: 92D30, 37N25, 35K51, 35Q92
Keywords and Phrases: COVID-19, SEI_cI_uR model, Anxiety
This research is supported by the DOST-ASTHRDP Grant

from lower socioeconomic backgrounds [14]. The findings highlight the disproportionate impact of mental health challenges on students from economically disadvantaged families, emphasizing the need for targeted support. The perception of vulnerability during a pandemic also varies significantly based on socioeconomic status, with individuals from lower-income backgrounds often experiencing heightened anxiety compared to their higher-income counterparts. Furthermore, social roles have been shown to influence anxiety levels, with certain groups facing unique stressors that exacerbate psychological distress. A broader study across multiple populations found notable variations in anxiety prevalence with healthcare workers exhibiting the highest levels [16]. These findings underscore the significant impact of anxiety across diverse groups, suggesting that mental health interventions can be effective in addressing this issue and warranting further investigation into the factors contributing to these disparities.

Despite the significant impact of mental well-being on health outcomes and public health responses, incorporating psychological behavior in the context of disease contagion models remains underexplored. This exigency to account for psychological parameters, specifically anxiety, in the context of the disease contagion model is herewith addressed.

The rest of the paper is organized as follows: Section 2 introduces the behavior-contagion model, detailing its key parameters and the derivation of the anxiety variable and contact rate expressions. Section 3 offers a comprehensive mathematical analysis of the model, demonstrating the existence of a unique solution, and proving its nonnegativity and boundedness. This section also examines the equilibrium points and their stability, and calculates the critical reproduction number, $\mathcal{R}_0$. Finally, Section 4 presents the simulation results, which are shown to be consistent with the theoretical predictions.

2 Mathematical model and its study

2.1 Description of the model

This study focuses on five components of the epidemic flow, comprising the densities of Susceptible individuals (S), Exposed individuals (E), Confirmed Infected individuals (I_c), Unreported Infected individuals (I_u), and Removed individuals (R). We anchor the formulation of our mathematical model on the standard strategy developed in the literature concerning the Susceptible-Infected-Recovered (SIR) model [6, 10]. Parallel to bygone epidemics, we assume everyone is susceptible to the disease prior to infection. Upon exposure to the pathogen, individuals enter the exposed compartment E and after the incubation period, transition into either the confirmed infected (I_c) or unreported infected (I_u) compartments. Susceptible individuals can only be infected by infected individuals and recovered individuals might get susceptible again over time. The flowchart of the model is shown in Figure 1.

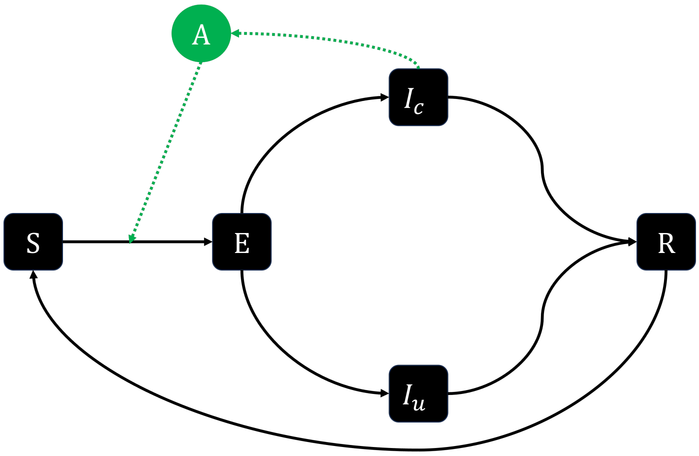

Figure 1: Compartmental representation of the SEI_cI_uR-model.

The dynamics is governed by a system of five (5) ordinary differential equations (ODE) as follows, for $t > 0$,

$$\begin{cases} S'(t) = -\omega(A)(\beta_c I_c + \beta_u I_u)\dfrac{S}{N} + \theta R \\ E'(t) = \omega(A)(\beta_c I_c + \beta_u I_u)\dfrac{S}{N} - \delta E \\ I_c'(t) = p\delta E - \gamma I_c \\ I_u'(t) = (1-p)\delta E - \gamma I_u \\ R'(t) = \gamma(I_c + I_u) - \theta R. \end{cases} \tag{1}$$

Each equation corresponds to a rate of a compartmental flow, where the contact parameter is described as a function of anxiety level A. The first term of the first equation corresponds to the flow of susceptibles to the exposed compartment and the second term constitutes the resusceptible population. The second term of the second equation corresponds to the proportion leaving the exposed compartment after a period of latency. The third and fourth equations signify the dynamics of the infected confirmed and unreported, respectively, based on the parameter $0 \leqslant p \leqslant 1$. Finally, the last equation corresponds to the dynamics of the recovered individuals, considering the tendency of reinfection. The forms of the anxiety variable and the contact parameter ω are detailed in the succeeding subsections. Assuming a constant living population N, we have

$$N = S + E + I_c + I_u + R.$$

No new recruit is added. The corresponding parameters are outlined in Table 1.

Table 1: Description of the parameters

Parameters	**Description**
$\omega(A)\beta_c$	Transmission rate from S to E from contact with I_c
$\omega(A)\beta_u$	Transmission rate from S to E from contact with I_u
δ	Latency rate
p	Probability to be confirmed infected
$(1-p$	Probability to be unreported infected
γ	Recovery rate
θ	Transmission rate from R to S

2.2 Derivation of the anxiety variable

In this study, we consider two forms of the anxiety variable. The first form is given by a simple linear model, where the rate of change of anxiety is based on the two main influences:

$$A'(t) = -\alpha_0(A - A_{eq}) + \frac{\alpha_1 I_c}{1 + \beta_1 I_c}. \tag{2}$$

The tendency for oscillations of the anxiety variable is modeled by the forced oscillation

$$A''(t) = \epsilon\alpha_0(1 - A^2)A'(t) - \alpha_0(A(t) - A_{eq}) + \frac{\alpha_1 I_c}{1 + \beta_1 I_c}. \tag{3}$$

Here, A_{eq} denotes the anxiety at equilibrium level. The nonlinearity and oscillations of anxiety around the equilibrium level depict more realistic human dynamics in response to a stimulus.

The Yerkes-Dodson law [21] applies to understanding the relationship between anxiety and behavior in the context of infectious diseases, COVID-19 in particular. We provide that our contact parameter follows the Yerkes-Dodson law described by normal distribution

$$\omega(A) = \frac{\omega_0}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(A-\mu)^2}{2\sigma^2}\right). \tag{4}$$

It is, thus, a proposition of the Yerkes-Dodson curve that there is an optimal level of anxiety that promotes adaptive behaviors in response to a threat, striking a balance between awareness and concern without succumbing to overwhelming anxiety.

3 Results

In this section, we provide the details for the system (1)-(2).

3.1 Global well-posedness

We need to ensure that the model's solution exists and is unique for any given initial conditions. Unrealistic results, such as negative populations or erratic behavior, undermine its usefulness for understanding the dynamics of disease progression. Thus, we establish that the solutions are well-defined and satisfy nonnegativity and boundedness.

Define

$$\Omega = \{(S, E, I_c, I_u, R, A) \in \mathbb{R}_+^6; 0 \leqslant S + E + I_c + I_u + R \leqslant N; 0 \leqslant A \leqslant A_{\max}\}.$$

In the succeeding results, we refer to the system of equations (1)−(2), unless otherwise specified.

Theorem 3.1. *Assuming that the initial condition lies in Ω, there exists a unique global-in-time solution (S, E, I_c, I_u, R, A) in $\mathscr{C}(\mathbb{R}_+; \Omega)$.*

Proof. Since the right-hand sides of the differential equations governing the system are continuous, with continuous partial derivatives in Ω, the Cauchy-Lipschitz theorem guarantees the existence of a unique solution on a short time interval. This establishes the local existence and uniqueness of the solution in $\mathscr{C}([0, T]; \Omega)$.

Next, we prove that the set Ω is positively invariant. This means that if the initial conditions lie in Ω, then the solution remains in Ω for all $t \geqslant 0$. Observe that if $S = 0$ and $R \geqslant 0$,

$$\frac{dS}{dt} = \theta R.$$

As all parameters are nonnegative, $\frac{dS}{dt} \geqslant 0$. This means S is increasing and $S(t) \geqslant 0$, for all $t > 0$. Similarly, if $E = 0$, then

$$\frac{dE}{dt} = \omega(A)(\beta_c I_c + \beta_u I_u)\frac{S}{N} \geqslant 0,$$

which implies E is increasing and $E(t) \geqslant 0$, for all $t > 0$. Also, if I_c, $I_u = 0$, then

$$\frac{dI_c}{dt} = p\delta E, \quad \frac{dI_u}{dt} = (1-p)\delta E \geqslant 0,$$

respectively. Thus, I_c, I_u are increasing and $I_c(t)$, $I_u(t) \geqslant 0$ for all $t > 0$. Moreover, if $R = 0$, then

$$\frac{dR}{dt} = \gamma(I_c + I_u) \geqslant 0,$$

so that $R(t) \geqslant 0$ for all $t > 0$. Finally, assuming $A = 0$,

$$\frac{dA}{dt} = \alpha_0 A_{eq} + \frac{\alpha_1 I_c}{1 + \beta_1 I_c} \geqslant 0,$$

which means $A(t) \geqslant 0$ for all $t > 0$. Since $S + E + I_c + I_u + R = N$, for all $t \geqslant 0$, the solution remains within the region Ω for all $t \geqslant 0$.

Define the total population N as the sum of the state variables:

$$N(t) = S(t) + E(t) + I_c(t) + I_u(t) + R(t).$$

Since each of the state variables $S(t), E(t), I_c(t), I_u(t), R(t)$ is nonnegative, and since $\frac{dN(t)}{dt} = 0$ so that $N = N(t) = N(0)$, for all time $t \geqslant 0$. This implies that the sum of the state variables is bounded by the initial total population, i.e.,

$$S(t) + E(t) + I_c(t) + I_u(t) + R(t) \leqslant N(0),$$

for all $t \geqslant 0$. Since each of these variables is non-negative, we have

$$0 \leqslant S(t), E(t), I_c(t), I_u(t), R(t) \leqslant N(0).$$

Therefore, each human compartment is uniformly bounded for all time $t \geqslant 0$ and since the anxiety variable is scaled and given to be $0 \leqslant A \leqslant A_{max}$, it is also bounded. This boundedness extends the local solution to a global solution on the interval $[0, \infty)$. Specifically, there exists a unique global-in-time solution in $\mathscr{C}(\mathbb{R}_+; \Omega)$. □

3.2 Basic reproduction number

The reproduction number $\mathcal{R}_0$ is a key metric in epidemiological modeling that indicates the average number of secondary infections generated by a single infected individual in a fully susceptible population [13]. It helps assess the potential for an outbreak to grow or decline—if $\mathcal{R}_0$ is greater than 1, the disease is likely to spread, while if $\mathcal{R}_0$ is less than 1, the outbreak will eventually die out. Understanding $\mathcal{R}_0$ allows public health agencies to design targeted interventions to control the spread of the disease. The reproduction number $\mathcal{R}_0$ can be computed thanks to the next generation matrix of the model as in [18]. Since the infected individuals are in E, I_c and I_u, the rate of appearance of new infections in each compartment $\mathcal{F}$ and the rate of other transitions between all compartments $\mathcal{V}$ can be rewritten as

$$\mathcal{F} = \begin{pmatrix} \omega(\beta_c I_c + \beta_u I_u)\frac{S}{N} \\ 0 \\ 0 \end{pmatrix}, \ \mathcal{V} = \begin{pmatrix} \delta E \\ \gamma I_c - p\delta E \\ \gamma I_u - (1-p)\delta E \end{pmatrix}.$$

Thus,

$$F = \begin{pmatrix} 0 & \frac{\omega\beta_c S}{N} & \frac{\omega\beta_u S}{N} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \delta & 0 & 0 \\ -p\delta & \gamma & 0 \\ -(1-p)\delta & 0 & \gamma \end{pmatrix}, \quad V^{-1} = \begin{pmatrix} \frac{1}{\delta} & 0 & 0 \\ \frac{p}{\gamma} & \frac{1}{\gamma} & 0 \\ \frac{1-p}{\gamma} & 0 & \frac{1}{\gamma} \end{pmatrix}.$$

Therefore, the next generation matrix is

$$FV^{-1} = \begin{pmatrix} \frac{p\omega\beta_c S}{\gamma N} + \frac{(1-p)\omega\beta_u S}{\gamma N} & \frac{\omega\beta_u S}{\gamma N} & \frac{\omega\beta_u S}{\gamma N} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

We deduce that the basic reproduction number is

$$\mathcal{R}_0 := \omega \left(\frac{p\beta_c}{\gamma} + \frac{(1-p)\beta_u}{\gamma} \right) \frac{S^*}{N^*}.$$

Biologically speaking, the first term $\frac{p\beta_c}{\gamma}$ represents the transmission due to confirmed individuals during the average infection period $1/\gamma$. The second one concerns the unreported cases.

3.3 Equilibrium behavior

Studying the equilibrium behavior of an epidemiological model is crucial because it helps identify long-term patterns in the spread of the disease, such as whether the infection will persist or eventually die out with time, which is what we wanted to achieve.

Since a steady state solution is a solution to the system that is constant for all time t, we set

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI_c}{dt} = \frac{dI_u}{dt} = \frac{dR}{dt} = \frac{dA}{dt} = 0.$$

Theorem 3.2. *The disease-free equilibrium (DFE) of the system is* $(N, 0, 0, 0, 0, A_{eq})$.

Proof. Let $(S^*, E^*, I_c^*, I_u^*, R^*, A^*)$ be a DFE point. Then $E^* = I_c^* = I_u^* = 0$ and

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI_c}{dt} = \frac{dI_u}{dt} = \frac{dR}{dt} = \frac{dA}{dt} = 0.$$

Thus, we have

$$\begin{aligned} \theta R^* &= 0 \\ -\theta R^* &= 0 \\ -\alpha_0(A^* - A_{eq}) &= 0. \end{aligned}$$

Given that all of the parameters are nonzero, we have $R^* = 0$ and $A^* = A_{eq}$. Evaluating the first equation (1) with this value, we have $\frac{dS}{dt} = 0$. Hence, $S(0) = S(t) = N$, $\forall t$, so that $S^* = N$. Therefore, the disease-free equilibrium point is given by $(N, 0, 0, 0, 0, A_{eq})$. □

Theorem 3.3. *The endemic equilibrium (EE) of the system is*

$$\Bigg(\frac{N}{\mathcal{R}_0}, \frac{\gamma\theta}{\gamma\theta + \delta\theta + \delta\gamma}\Big(1 - \frac{1}{\mathcal{R}_0}\Big)N, \frac{p\delta\theta}{\gamma\theta + \delta\theta + \delta\gamma}\Big(1 - \frac{1}{\mathcal{R}_0}\Big)N,$$

$$\frac{(1-p)\delta\theta}{\gamma\theta + \delta\theta + \delta\gamma}\Big(1 - \frac{1}{\mathcal{R}_0}\Big)N, \frac{\delta\gamma}{\gamma\theta + \delta\theta + \delta\gamma}\Big(1 - \frac{1}{\mathcal{R}_0}\Big)N, A_{eq} + \frac{\alpha_1 I_c^*}{\alpha_0(1 + \beta_1 I_c^*)} \Bigg).$$

Proof. Let $(S^*, E^*, I_c^*, I_u^*, R^*, A^*)$ be an EE point. Then

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI_c}{dt} = \frac{dI_u}{dt} = \frac{dR}{dt} = \frac{dA}{dt} = 0.$$

Thus, we have

$$\begin{aligned}
0 &= -\omega(\beta_c I_c^* + \beta_u I_u^*)\frac{S^*}{N} + \theta R^* && (5)\\
0 &= \omega(\beta_c I_c^* + \beta_u I_u^*)\frac{S^*}{N} - \delta E^* && (6)\\
0 &= p\delta E^* - \gamma I_c^* && (7)\\
0 &= (1-p)\delta E^* - \gamma I_u^* && (8)\\
0 &= \gamma(I_c^* + I_u^*) - \theta R^* && (9)\\
0 &= -\alpha_0(A^* - A_{eq}) + \frac{\alpha_1 I_c^*}{1+\beta_1 I_c^*}. && (10)
\end{aligned}$$

The anxiety equation (10) yields $A^* = A_{eq} + \frac{\alpha_1 I_c^*}{\alpha_0(1+\beta_1 I_c^*)}$. For the human compartments, adding equations (5) and (6) yields $R^* = \frac{\delta E^*}{\theta}$. From equations (7) and (8), we get $I_c^* = \frac{p\delta E^*}{\gamma}$ and $I_u^* = \frac{(1-p)\delta E^*}{\gamma}$, respectively. Substituting these values to equation (6), we have

$$\omega\left(\beta_c\left(\frac{p\delta E^*}{\gamma}\right) + \beta_u\left(\frac{(1-p)\delta E^*}{\gamma}\right)\right)\frac{S^*}{N} - \delta E^* = 0,$$

so that

$$\omega\big(p\beta_c + (1-p)\beta_u\big)S^* = \gamma N.$$

Hence,

$$S^* = \frac{\gamma N}{\omega\big(p\beta_c + (1-p)\beta_u\big)}.$$

The remaining E^*, I_c^*, I_u^*, R^* can be obtained using the population constraint

$$S^* + E^* + I_c^* + I_u^* + R^* = N.$$

Substituting the values obtained above to solve for E^*, we have

$$S^* + E^* + \frac{p\delta E^*}{\gamma} + \frac{(1-p)\delta E^*}{\gamma} + \frac{\delta E^*}{\theta} = N.$$

This gives

$$S^* + E^*\left(1 + \frac{p\delta}{\gamma} + \frac{(1-p)\delta}{\gamma} + \frac{\delta}{\theta}\right) = N.$$

Let $K = 1 + \frac{p\delta}{\gamma} + \frac{(1-p)\delta}{\gamma} + \frac{\delta}{\theta}$. Then, $S^* = N - E^*K$ and so

$$\begin{aligned}
N - E^*K &= \frac{\gamma N}{\omega\big(p\beta_c+(1-p)\beta_u\big)}\\
\Rightarrow E^*K &= \frac{N\Big(\omega\big(p\beta_c+(1-p)\beta_u\big)-\gamma\Big)}{\omega\big(p\beta_c+(1-p)\beta_u\big)}\\
\Rightarrow E^* &= \frac{N\Big(\omega\big(p\beta_c+(1-p)\beta_u\big)-\gamma\Big)}{K\Big(\omega\big(p\beta_c+(1-p)\beta_u\big)\Big)} = \frac{N\Big(\omega\big(p\beta_c+(1-p)\beta_u\big)-\gamma\Big)}{\Big(1+\frac{p\delta}{\gamma}+\frac{(1-p)\delta}{\gamma}+\frac{\delta}{\theta}\Big)\Big(\omega\big(p\beta_c+(1-p)\beta_u\big)\Big)}\\
&= \frac{\gamma\theta}{\gamma\theta+\delta\theta+\delta\gamma}\left(1-\frac{1}{\mathcal{R}_0}\right)N.
\end{aligned}$$

Hence,

$$
\begin{array}{rcl}
I_c^* & = & \frac{p\delta E^*}{\gamma} = \frac{p\delta\theta}{\gamma\theta+\delta\theta+\delta\gamma}\left(1-\frac{1}{\mathcal{R}_0}\right)N \\
I_u^* & = & \frac{(1-p)\delta E^*}{\gamma} = \frac{(1-p)\delta\theta}{\gamma\theta+\delta\theta+\delta\gamma}\left(1-\frac{1}{\mathcal{R}_0}\right)N \\
R^* & = & \frac{\delta E^*}{\theta} = \frac{\delta\gamma}{\gamma\theta+\delta\theta+\delta\gamma}\left(1-\frac{1}{\mathcal{R}_0}\right)N \\
S^* & = & \frac{\gamma N}{\omega\left(p\beta_c+(1-p)\beta_u\right)} = \frac{N}{\mathcal{R}_0}.
\end{array}
$$

□

The stability of the equilibrium point can be performed by calculating the roots of the characteristic polynomial

$$\det(J^* - \lambda I) = 0$$

where J^* is the Jacobian matrix evaluated at the equilibrium and I is the identity matrix. Concerning the endemic equilibrium, we used the software *Mathematica* to derive the coefficients of the characteristic polynomial:

$$P(\lambda) = \lambda^6 + a_1\lambda^5 + a_2\lambda^4 + a_3\lambda^3 + a_4\lambda^2 + a_5\lambda + a_6.$$

Theorem 3.4. *1. If $\mathcal{R}_0 \leqslant 1$, the disease free equilibrium is locally asymptotically stable.*

2. If $\mathcal{R}_0 > 1$, the DFE is unstable and the endemic equilibrium is locally stable if the system follows the Routh-Hurwitz criteria and unstable otherwise.

Proof. By computing the eigenvalues of the Jacobian matrix, we deduce that if $\mathcal{R}_0 \leqslant 1$, then the DFE is locally asymptotically stable and unstable whenever $\mathcal{R}_0 > 1$.

With the help of the Routh-Hurwitz criteria, we deduce that all eigenvalues of $P(\lambda)$ are negative or have negative real parts if and only if $a_j > 0, j = 1, ..., 6$ and the determinant of all Hurwitz matrices are positive, that is,

$$\det \mathrm{H}_1 = \mathrm{a}_1 > 0,$$

$$\det \mathrm{H}_2 = \det \begin{bmatrix} a_1 & 1 \\ a_3 & a_2 \end{bmatrix} > 0,$$

$$\det \mathrm{H}_3 = \det \begin{bmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{bmatrix} > 0,$$

$$\det \mathrm{H}_4 = \det \begin{bmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 \\ a_5 & a_4 & a_3 & a_2 \\ 0 & a_6 & a_5 & a_4 \end{bmatrix} > 0,$$

$$\det \mathrm{H}_5 = \det \begin{bmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \\ 0 & a_6 & a_5 & a_4 & a_3 \\ 0 & 0 & 0 & a_6 & a_5 \end{bmatrix} > 0,$$

$$\det \mathrm{H}_6 = \det \begin{bmatrix} a_1 & 1 & 0 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & 1 \\ 0 & a_6 & a_5 & a_4 & a_3 & a_2 \\ 0 & 0 & 0 & a_6 & a_5 & a_4 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{bmatrix} > 0.$$

If the Routh-Hurwitz criterion is satisfied, then the endemic equilibrium is locally stable (see Appendix A for the values of $a_j > 0, j = 1, ..., 6$). $\square$

In the remaining results, we consider the disease-free equilibrium for the oscillatory anxiety equation (3). One set $A'(t) = B(t)$ so that $B'(t) = A''(t)$. Hence, we have the transformed system

$$\begin{aligned} A' &= B \\ B' &= \epsilon\alpha_0(1 - A^2)B - \alpha_0(A - A_{eq}) + \frac{\alpha_1 I_c}{1 + \beta_1 I_c}. \end{aligned}$$

Therefore, along with equations (1) − (3), we have the new system of seven first-order ODEs:

$$\begin{cases} \dfrac{dS}{dt} = -\omega(A)(\beta_c I_c + \beta_u I_u)\dfrac{S}{N} + \theta R \\ \dfrac{dE}{dt} = \omega(A)(\beta_c I_c + \beta_u I_u)\dfrac{S}{N} - \delta E \\ \dfrac{dI_c}{dt} = p\delta E - \gamma I_c \\ \dfrac{dI_u}{dt} = (1 - p)\delta E - \gamma I_u \\ \dfrac{dR}{dt} = \gamma(I_c + I_u) - \theta R \\ \dfrac{dA}{dt} = B \\ \dfrac{dB}{dt} = \epsilon\alpha_0(1 - A^2)B - \alpha_0(A - A_{eq}) + \dfrac{\alpha_1 I_c}{1 + \beta_1 I_c}. \end{cases} \tag{11}$$

Theorem 3.5. *If $\mathcal{R}_0 \leqslant 1$, the DFE $(N, 0, 0, 0, 0, A_{eq}, 0)$ of the system (11) is locally asymptotically stable and unstable otherwise.*

Proof. As in Theorem 3.2 and setting $\frac{dA}{dt} = \frac{dB}{dt} = 0$, we have $A^* = A_{eq}$ and $B^* = 0$ so that we have the DFE $(N, 0, 0, 0, 0, A_{eq}, 0)$. Moreover, by computing the eigenvalues of the Jacobian matrix, we deduce that the DFE point is locally asymptotically stable whenever $\mathcal{R}_0 \leqslant 1$. Otherwise, the DFE is unstable. $\square$

4 Numerical Simulations

Numerical simulations illustrate theoretical results by providing a computational approach to check the accuracy of analytical solutions. This section considers testing the model under varying conditions and ensuring the theoretical predictions align with the observed behavior.

We fixed the initial conditions of the compartments as follows :

$$S(0) = 300000, E(0) = 60000, I_c(0) = 30000, I_u(0) = 20000, R(0) = 0, N = 410000.$$

We also fixed assumed values of the parameters and some based on literature [2] as follows:

$$\gamma = 0.103809,\ \theta = 1.49346 \times 10^{-2},\ \alpha_0 = 2.2412 \times 10^{-2},\ \alpha_1 = 4.331216 \times 10^{-3},$$

$$\beta_1 = 0.19676,\ \omega_0 = 0.99994,\ \delta = 0.269925,\ p = 0.387702.$$

For anxiety metrics, we set

$$\sigma = 2.01,\ \mu = 18.664.$$

The infection rates β_c and β_u are varying allowing to consider the scenarios of Theorem 3.2 where $\mathcal{R}_0 \leqslant 1$ and $\mathcal{R}_0 > 1$. The final time of the simulation is $T = 365$. Table 2 summarizes the results, and visualizations of the compartments are illustrated in the succeeding figures.

	$\beta_c = 0.05, \beta_u = 0.025$	$\beta_c = 3, \beta_u = 5$
$\mathcal{R}_0$	$0.0663 < 1$	$8.08 > 1$
$S(T)$	409411.69	57202.73
$E(T)$	0	16277.99
$I_c(T)$	0	16409.97
$I_u(T)$	0	25916.28
$R(T)$	588.31	294193.03
$A(T)$	18.67	19.65

Table 2: Values of the model parameters for a final time $T = 365$.

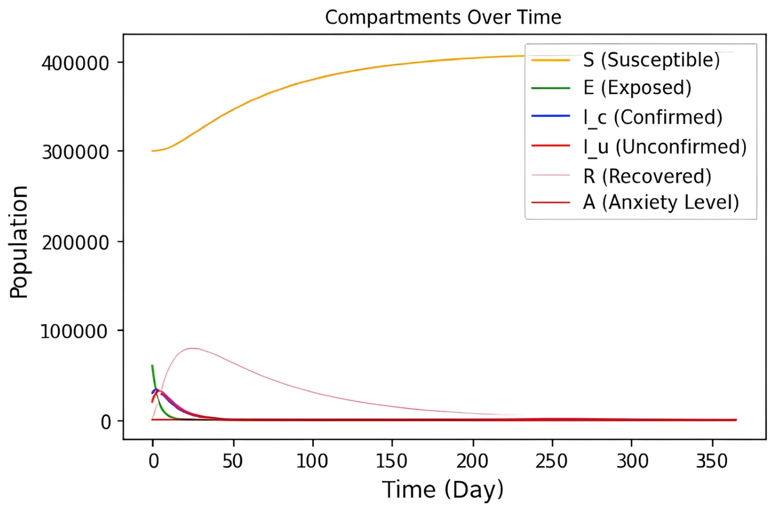

Figure 2: $\beta_c = 0.05, \beta_u = 0.025$ with $\mathcal{R}_0 = 0.0663 < 1$

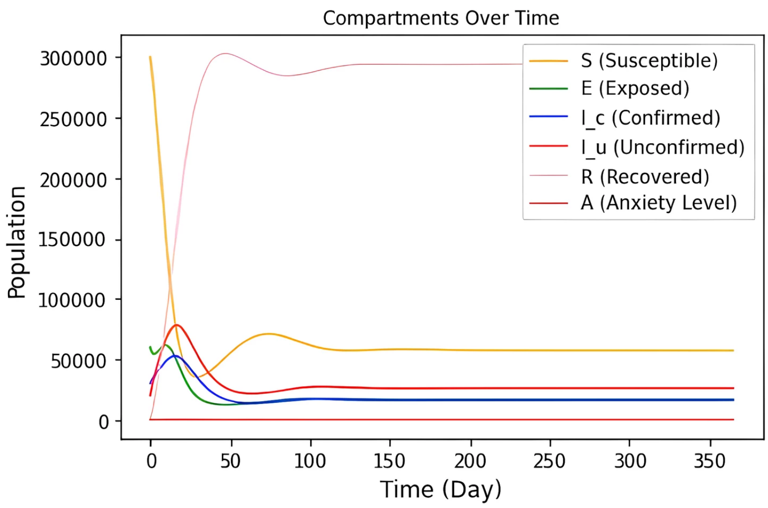

Figure 3: $\beta_c = 3, \beta_u = 5$ with $\mathcal{R}_0 = 8.077 > 1$

Increasing final time $T = 1000$, compartment values eventually converge to the predicted equilibrium point $(N, 0, 0, 0, 0, A_{eq})$. Figure 4 visualizes this result.

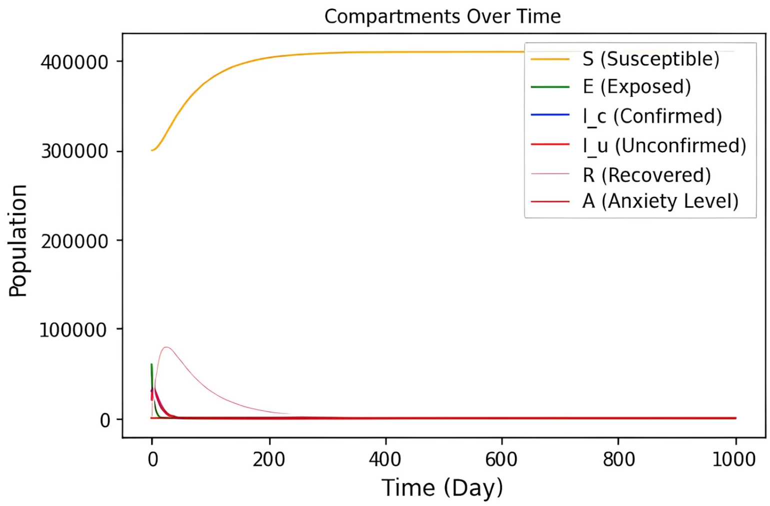

Figure 4: $\beta_c = 0.05, \beta_u = 0.025$ with $\mathcal{R}_0 = 0.07 < 1$

On the other hand, increasing the final time $T = 5000$ for the case where $\beta_c = 3, \beta_u = 5$ still yields $\mathcal{R}_0 > 1$, so that infections continue to circulate and the solution converges to the EE. Figure 5 visualizes this result.

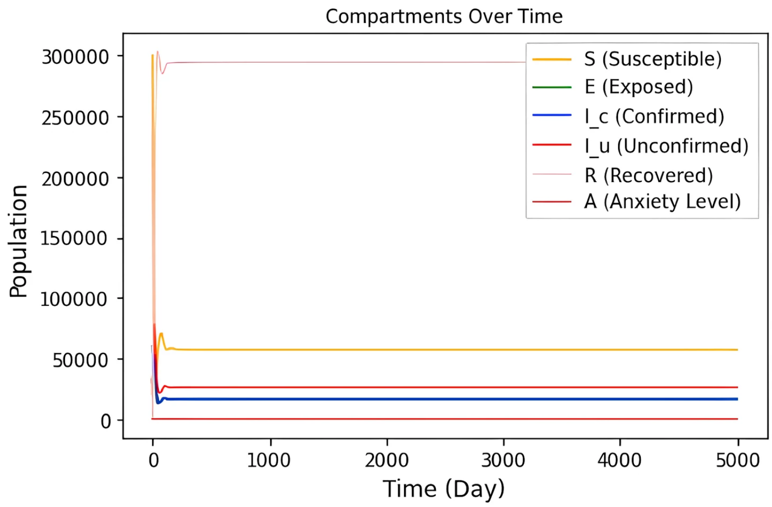

Figure 5: $\beta_c = 3, \beta_u = 5, R_0 = 8.077$

Finally, for Theorem 3.5 and oscillatory anxiety, we fixed the same initial conditions and parameters are set as $\beta_c = 0.25$, $\beta_u = 0.07$, $\theta = 0.3$, $\alpha_0 = 0.0125$, $\alpha_1 = 0.333$, $\beta_1 = 0.5$, $\omega_0 = 0.4$, $\epsilon = 0.00005$, and $B_0 = 3$. We set the final time of our simulations to $T = 1000$.

https://doi.org/10.62071/tmjm.v6i2.718

	$\beta_c = 0.25, \beta_u = 0.07$	$\beta_c = 4, \beta_u = 7$
$\mathcal{R}_0$	$0.1069 < 1$	$4.7604 > 1$
$S(T)$	409999.99	409492.458
$E(T)$	0	115.177
$I_c(T)$	0	117.387
$I_u(T)$	0	185.389
$R(T)$	0	89.586
$A(T)$	18.664	25.611
Predicted A^*	18.664	18.664
$B(T)$	0	1.3244
Predicted B^*	0	0

Table 3: Values of the model compartments at $T = 1000$

Table 3 shows that the values of all compartments stabilize around the DFE.

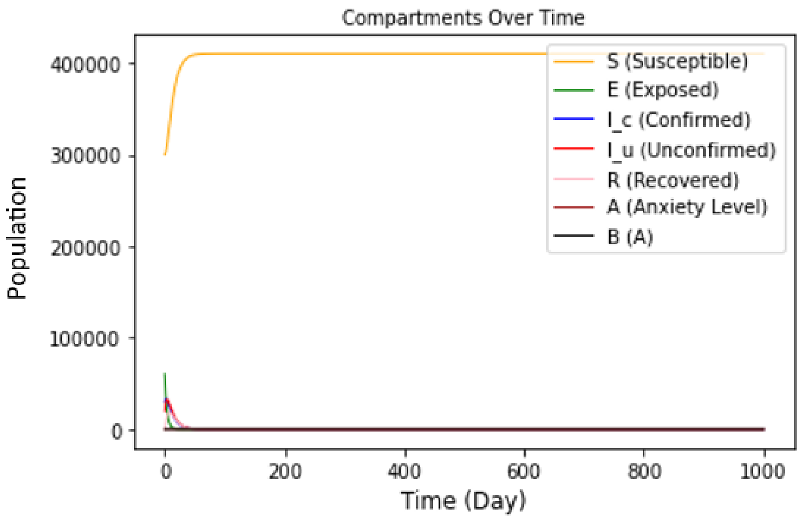

Figure 6: $\beta_c = 0.25$, $\beta_u = 0.07$, $\epsilon = 0.00005$, and $\mathcal{R}_0 = 0.1069$.

Here, increasing ϵ results in a lower oscillation of the anxiety parameter. We now consider increasing infection rates $\beta_c = 4$ and $\beta_u = 7$ with a remarkably low $\epsilon = 0.00005$.

Compartment	Value
$S(T)$	409492.458
$E(T)$	115.177
$I_c(T)$	117.387
$I_u(T)$	185.389
$R(T)$	89.586
$A(T)$	25.611
Predicted A^*	18.664
$B(T)$	1.3244
Predicted B^*	0

Table 4: Values of the model compartments at $T = 1000$

Observe from Table 4 that the compartments do not stabilize around the DFE whenever $\mathcal{R}_0 > 1$. Figure 7 illustrates this trend over time.

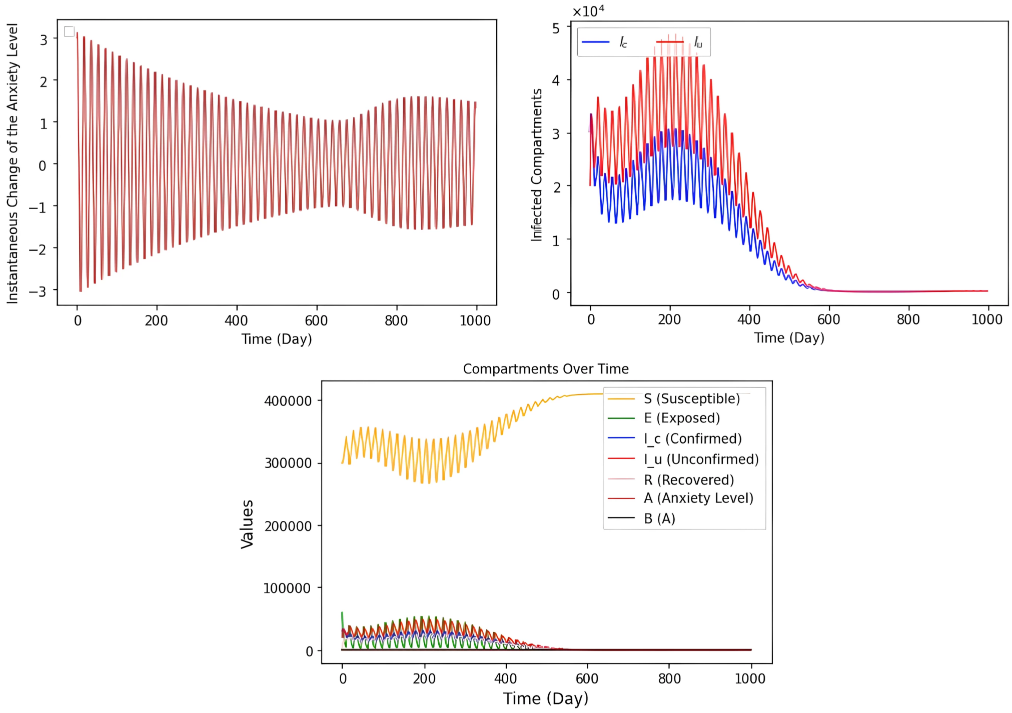

Figure 7: $\beta_c = 4$, $\beta_u = 7$, $\epsilon = 0.00005$, and $\mathcal{R}_0 = 4.7604$. The upper left figure shows high oscillations of anxiety, the upper right figure shows the trend of the infected compartments I_c and I_u, and the bottom figure shows the values of all the compartments (anxiety level and populations) after the final time of the simulations.

Varying the value of ϵ provides differing levels of oscillations of the compartments over time.

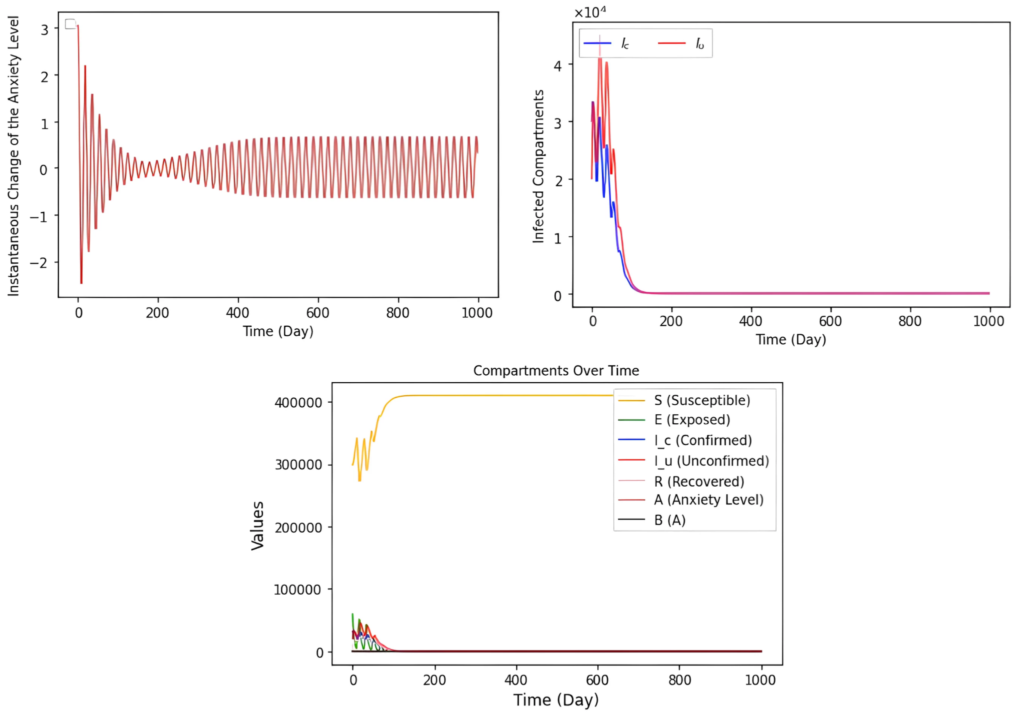

Figure 8: $\beta_c = 4$, $\beta_u = 7$, $\epsilon = 0.005$, and $\mathcal{R}_0 = 4.7604$. The same figure descriptions apply from Figure 7, this time with low ϵ.

This emphasizes the significant impact of an oscillating anxiety variable in the dynamics of COVID-19 disease progression. In contrast with the above highly oscillatory trend, the following simulations fixed the same values, except $\epsilon = 0.005$.

A Appendix

Cont. Proof of Theorem 3.1(2)

Using the online software Mathematica, it is obtained that

$$\begin{aligned}a_1 = \Big(&2N^2\gamma + N^2\delta + N^2\theta - N^2R\theta + N^2\alpha_0 + 4N^2I_c\gamma\beta_1 + 2N^2I_c\delta\beta_1 +\\ &2N^2I_c\theta\beta_1 - 2N^2RI_c\theta\beta_1 + 2N^2I_c\alpha_0\beta_1 + 2N^2I_c^2\gamma\beta_1^2 + N^2I_c^2\delta\beta_1^2 +\\ &N^2I_c^2\theta\beta_1^2 - N^2RI_c^2\theta\beta_1^2 + N^2I_c^2\alpha_0\beta_1^2 + NSI_c\omega(A)\beta_c +\\ &2NSI_c^2\omega(A)\beta_1\beta_c + NSI_c^3\omega(A)\beta_1^2\beta_c + NI_uS\omega(A)\beta_u +\\ &2NI_uSI_c\omega(A)\beta_1\beta_u + NI_uSI_c^2\omega(A)\beta_1^2\beta_u\Big) \Big/ N^2\left(1 + I_c\beta_1\right)^2\end{aligned}$$

$$\begin{aligned}a_2 = &N^2\gamma^2 + 2N^2\gamma\delta + 2N^2\gamma\theta - 2N^2R\gamma\theta + N^2\delta\theta - N^2R\delta\theta - N^2R\theta^2 +\\ &2N^2\gamma\alpha_0 + N^2\delta\alpha_0 + N^2\theta\alpha_0 - N^2R\theta\alpha_0 + 2N^2I_c\gamma^2\beta_1 + 4N^2I_c\gamma\delta\beta_1 +\\ &4N^2I_c\gamma\theta\beta_1 - 4N^2RI_c\gamma\theta\beta_1 + 2N^2I_c\delta\theta\beta_1 - 2N^2RI_c\delta\theta\beta_1 - 2N^2RI_c\theta^2\beta_1 +\\ &4N^2I_c\gamma\alpha_0\beta_1 + 2N^2I_c\delta\alpha_0\beta_1 + 2N^2I_c\theta\alpha_0\beta_1 - 2N^2RI_c\theta\alpha_0\beta_1 + N^2I_c^2\gamma^2\beta_1^2 +\\ &2N^2I_c^2\gamma\delta\beta_1^2 + 2N^2I_c^2\gamma\theta\beta_1^2 - 2N^2RI_c^2\gamma\theta\beta_1^2 + N^2I_c^2\delta\theta\beta_1^2 - N^2RI_c^2\delta\theta\beta_1^2 -\\ &N^2RI_c^2\theta^2\beta_1^2 + 2N^2I_c^2\gamma\alpha_0\beta_1^2 + N^2I_c^2\delta\alpha_0\beta_1^2 + N^2I_c^2\theta\alpha_0\beta_1^2 - N^2RI_c^2\theta\alpha_0\beta_1^2 -\\ &Np_dX\omega(A)\beta_e + 2NSI_c\gamma\omega(A)\beta_c + NSI_c\delta\omega(A)\beta_c + NSI_c\theta\omega(A)\beta_c +\\ &NSI_c\omega(A)\alpha_0\beta_c - 2NpSI_c\omega(A)\beta_1\beta_c + 4NSI_c^2\gamma\omega(A)\beta_1\beta_c +\\ &2NSI_c^2\delta\omega(A)\beta_1\beta_c + 2NSI_c^2\theta\omega(A)\beta_1\beta_c + 2NSI_c^2\omega(A)\alpha_0\beta_1\beta_c +\\ &2NI_uSI_c\gamma\omega(A)\beta_u - NS\delta\omega(A)\beta_u + NpS\delta\omega(A)\beta_u + NI_uS\delta\omega(A)\beta_u +\\ &NI_uS\theta\omega(A)\beta_u + NI_uS\omega(A)\alpha_0\beta_u + 4NI_uSI_c\gamma\omega(A)\beta_1\beta_u -\\ &2NSI_c\delta\omega(A)\beta_1\beta_u + 2NpSI_c\delta\omega(A)\beta_1\beta_u + 2NI_uSI_c\delta\omega(A)\beta_1\beta_u +\\ &2NI_uSI_c\theta\omega(A)\beta_1\beta_u + 2NI_uSI_c\omega(A)\alpha_0\beta_1\beta_u + 2NI_uSI_c^2\gamma\omega(A)\beta_1^2\beta_u -\\ &NSI_c^2\delta\omega\beta_1^2\beta_u + NpSI_c^2\delta\omega(A)\beta_1^2\beta_u + NI_uSI_c^2\delta\omega(A)\beta_1^2\beta_u +\\ &NI_uSI_c^2\theta\omega(A)\beta_1^2\beta_u + NI_uSI_c^2\omega(A)\alpha_0\beta_1^2\beta_u\Big) \Big/ N^2\left(1 + I_c\beta_1\right)^2,\end{aligned}$$

$$
\begin{aligned}
a_3 =\Big(& N^2S\gamma\delta - N^2pS\gamma\delta + N^2\gamma^2\delta + N^2\gamma^2\theta - N^2R\gamma^2\theta + 2N^2\gamma\delta\theta - 2N^2R\gamma\delta\theta - \\
& 2N^2R\gamma\theta^2 - N^2R\delta\theta^2 + N^2\gamma^2\alpha_0 + 2N^2\gamma\delta\alpha_0 + 2N^2\gamma\theta\alpha_0 - 2N^2R\gamma\theta\alpha_0 + \\
& N^2\delta\theta\alpha_0 - N^2R\delta\theta\alpha_0 - N^2R\theta^2\alpha_0 + N^2p\delta x\alpha_1 + 2N^2SI_c\gamma\delta\beta_1 - \\
& 2N^2pSI_c\gamma\delta\beta_1 + 2N^2I_c\gamma^2\delta\beta_1 + 2N^2I_c\gamma^2\theta\beta_1 - 2N^2RI_c\gamma^2\theta\beta_1 + \\
& 4N^2I_c\gamma\delta\theta\beta_1 - 4N^2RI_c\gamma\delta\theta\beta_1 - 4N^2RI_c\gamma\theta^2\beta_1 - 2N^2RI_c\delta\theta^2\beta_1 + \\
& 2N^2I_c\gamma^2\alpha_0\beta_1 + 4N^2I_c\gamma\delta\alpha_0\beta_1 + 4N^2I_c\gamma\theta\alpha_0\beta_1 - 4N^2RI_c\gamma\theta\alpha_0\beta_1 + \\
& 2N^2I_c\delta\theta\alpha_0\beta_1 - 2N^2RI_c\delta\theta\alpha_0\beta_1 - 2N^2RI_c\theta^2\alpha_0\beta_1 + N^2xI_c^2\gamma\delta\beta_1^2 - \\
& N^2pSI_c^2\gamma\delta\beta_1^2 + N^2I_c^2\gamma^2\delta\beta_1^2 + N^2I_c^2\gamma^2\theta\beta_1^2 - N^2RI_c^2\gamma^2\theta\beta_1^2 + \\
& 2N^2I_c^2\gamma\delta\theta\beta_1^2 - 2N^2RI_c^2\gamma\delta\theta\beta_1^2 - 2N^2RI_c^2\gamma\theta^2\beta_1^2 - 2N^2RI_c^2\delta\theta^2\beta_1^2 + \\
& N^2p\delta x\beta_c - NSI_c\gamma^2\omega\beta_c + 2NSI_c\gamma\delta\omega\beta_c - Np\delta SI_c\theta\omega\beta_c + 2NpI_cS\theta\omega\beta_c + \\
& 2NI_cS\gamma\theta\omega\beta_1 + NSI_c\delta\theta\omega\beta_c - NpI_cS\omega\alpha_0\beta_c + 2NSI_c\gamma\omega\alpha_0\beta_1 + \\
& NSI_c\delta\omega\alpha_0\beta_1 + NSI_c\theta\omega\alpha_0\beta_1 - 2NpI_cS\gamma\omega\beta_1 + 2NSI_cI_u^2\gamma^2\omega\beta_1 - \\
& 2NpI_cS\theta\omega\beta_1 + 2NpRI_cSI_u\theta\omega\beta_1N^2S\gamma\delta - N^2pS\gamma\delta + N^2\gamma^2\delta + N^2\gamma^2\theta - \\
& N^2R\gamma^2\theta + 2N^2\gamma\delta\theta - 2N^2R\gamma\delta\theta - 2N^2R\gamma\theta^2 - N^2R\delta\theta^2 + N^2\gamma^2\alpha_0 + \\
& 2N^2\gamma\delta\alpha_0 + 2N^2\gamma\theta\alpha_0 - 2N^2R\gamma\theta\alpha_0 + N^2\delta\theta\alpha_0 - N^2R\delta\theta\alpha_0 - N^2R\theta^2\alpha_0 + \\
& N^2p\delta S\beta_c + 2N^2SI_c\gamma\delta\beta_1 - 2N^2pSI_c\gamma\delta\beta_1 + 2N^2I_c\gamma^2\delta\beta_1 + 2N^2I_c\gamma^2\theta\beta_1 - \\
& 2N^2RI_c\gamma^2\theta\beta_1 + 4N^2I_c\gamma\delta\theta\beta_1 - 4N^2RI_c\gamma\delta\theta\beta_1 - 4N^2RI_c\gamma\theta^2\beta_1 - \\
& 2N^2RI_c\delta\theta^2\beta_1 + 2N^2I_c\gamma^2\alpha_0\beta_1 + 4N^2I_c\gamma\delta\alpha_0\beta_1 + 4N^2I_c\gamma\theta\alpha_0\beta_1 - \\
& 4N^2RI_c\gamma\theta\alpha_0\beta_1 + 2N^2I_c\delta\theta\alpha_0\beta_1 - 2N^2RI_c\delta\theta\alpha_0\beta_1 - 2N^2RI_c\theta^2\alpha_0\beta_1 + \\
& N^2SI_c^2\gamma\delta\beta_1^2 - N^2pSI_c^2\gamma\delta\beta_1^2 + N^2I_c^2\gamma^2\delta\beta_1^2 + N^2I_c^2\gamma^2\theta\beta_1^2 - N^2RI_c^2\gamma^2\theta\beta_1^2 + \\
& 2N^2I_c^2\gamma\delta\theta\beta_1^2 - 2N^2RI_c^2\gamma\delta\theta\beta_1^2 - 2N^2RI_c^2\gamma\theta^2\beta_1^2 + 2N^2RI_c^2\delta\theta^2\beta_1^2 + N^2p\delta S\beta_c - \\
& NSI_c\gamma^2\omega\beta_c + 2NSI_c\gamma\delta\omega\beta_c - Np\delta SI_c\theta\omega\beta_c + 2NpI_cS\theta\omega\beta_c + 2NI_cS\gamma\theta\omega\beta_1 + \\
& NSI_c\delta\theta\omega\beta_c - NpI_cS\omega\alpha_0\beta_c + 2NSI_c\gamma\omega\alpha_0\beta_1 + NSI_c\delta\omega\alpha_0\beta_1 + NSI_c\theta\omega\alpha_0\beta_1 - \\
& 2NpI_cSI_u\gamma\omega\beta_1 + 2NSI_cI_u^2\gamma^2\omega\beta_1 - 2NpI_cS\theta\omega\beta_1 + 2NpRI_cSI_u\theta\omega\beta_1\Big) \Big/ \\
& N^2\left(1 + I_c\beta_1\right)^2,
\end{aligned}
$$

$$
\begin{aligned}
a_4 = {} & N^2S\gamma^2\delta - N^2pS\gamma^2\delta - N^2p\delta S\gamma\theta - N^2RS\gamma\delta\theta + N^2pRS\gamma\delta\theta + N^2\gamma_2\delta\theta \\
& - N^2R\gamma^2\delta\theta - N^2R\gamma\theta^2 - 2N^2R\gamma\delta\theta^2 + N^2\gamma^2\delta\alpha_0 + N^2\gamma^2\theta\alpha_0 \\
& - N^2R\gamma_2\theta\alpha_0 + 2N^2\gamma\delta\theta\alpha_0 - 2N^2R\gamma\delta\theta\alpha_0 - 2N^2R\gamma\theta^2\alpha_0 \\
& - N^2R\delta\theta^2\alpha_0 + N^2p\delta S\gamma\alpha_1 + N^2p\delta S\theta\alpha_1 - N^2p\delta RS\theta\alpha_1 \\
& + 2N^2SI_c\gamma^2\delta\beta_1 - 2N^2pSI_c\gamma^2\delta\beta_1 - 2N^2p\delta SI_c\gamma\theta\beta_1 \\
& - 2N^2RSI_c\gamma\delta\theta\beta_1 + 2N^2pRSI_c\gamma\delta\theta\beta_1 + 2N^2I_c\gamma^2\delta\theta\beta_1 \\
& - 2N^2RI_c\gamma_2\delta\theta\beta_1 - 2N^2RI_c\gamma\theta^2\beta_1 - 4N^2RI_c\gamma\delta\theta^2\beta_1 \\
& + 2N^2I_c\gamma_2\delta\alpha_0\beta_1 + 2N^2I_c\gamma^2\theta\alpha_0\beta_1 - 2N^2RI_c\gamma^2\theta\alpha_0\beta_1 \\
& + 4N^2I_c\gamma\delta\theta\alpha_0\beta_1 - 4N^2RI_c\gamma\delta\theta\alpha_0\beta_1 - 4N^2RI_c\gamma\theta^2\alpha_0\beta_1
\end{aligned}
$$

$$
\begin{aligned}
&- 2N^2RI_c\delta\theta^2\alpha_0\beta_1 + N^2SI_c\gamma^2\delta\omega(A)\beta_c - Np\delta S\gamma\theta\omega(A)\beta_c \\
&+ Np\delta RS\gamma\theta\omega(A)\beta_c - Np\delta SI_c\gamma\theta\omega(A)\beta_c + NSI_c\gamma^2\theta\omega(A)\beta_c \\
&+ NSI_c\gamma\delta\theta\omega\beta_c + NpSI_c\gamma\delta\theta\omega(A)\beta_c + Np\delta RS\theta^2\omega(A)\beta_c \\
&- Np\delta S\gamma\omega(A)\alpha_0\beta_e + NSI_c\gamma^2\omega(A)\alpha_0\beta_c + 2NSI_c\gamma\delta\omega(A)\alpha_0\beta_c \\
&- Np\delta S\theta\omega(A)\alpha_0\beta_c + Np\delta RS\theta\omega(A)\alpha_0\beta_c + 2NSI_c\gamma\theta\omega\alpha_0\beta_c \\
&+ NSI_c\delta\theta\omega(A)\alpha_0\beta_c + 2NSI_c\delta d\theta\omega(A)\alpha_0\beta_c + 2NSI_c\delta^2 d\theta\omega(A)\beta_1 \\
&+ NSI_c\delta^2\delta\theta\beta_1 + 2NI_uS\gamma\delta\omega(A)\alpha_0\beta_u + 2NI_uS\gamma\delta\omega(A)\alpha_0\beta_u \\
&- NS\delta\theta\omega(A)\alpha_0\beta_u + NpS\delta\theta\omega(A)\alpha_0\beta_u + NRS\delta\theta\omega(A)\alpha_0\beta_u \\
&- NpRS\delta\theta\omega(A)\alpha_0\beta_u + NI_uS\delta\theta\omega(A)\alpha_0\beta_u + 2NI_uSI_c\gamma^2\delta\omega\beta_1\beta_u \\
&- 2Np\delta I_uSI_c\gamma\theta\omega(A)\beta_1\beta_u + 2NI_uSI_c\gamma^2\theta\omega(A)\beta_1\beta_u \\
&- 2NSI_c\gamma\delta\theta\omega(A)\beta_1\beta_u + 2NpSI_c\gamma\delta\theta\omega(A)\beta_1\beta_u \\
&+ 2NRSI_c\gamma\delta\theta\omega(A)\beta_1\beta_u - 2NpRSI_c\gamma\delta\theta\omega(A)\beta_1\beta_u \\
&+ 2NI_uSI_c\gamma\delta\theta\omega(A)\beta_1\beta_u + 2NpI_uSI_c\gamma\delta\theta\omega(A)\beta_1\beta_u \\
&+ 2NRSI_c\delta\theta^2\omega(A)\beta_1\beta_u - 2NpRSI_c\delta\theta^2\omega(A)\beta_1\beta_u \\
&+ 2NI_uSI_c\gamma^2\omega(A)\alpha_0\beta_1\beta_u - 2NSI_c\gamma\delta\omega(A)\alpha_0\beta_1\beta_u \\
&+ 2NpSI_c\gamma\delta\omega(A)\alpha_0\beta_1\beta_u + 4NI_uSI_c\gamma\delta\omega(A)\alpha_0\beta_1\beta_u \\
&+ 4NI_uSI_c\gamma\theta\omega(A)\alpha_0\beta_1\beta_u - 2NSI_c\delta\theta\omega(A)\alpha_0\beta_1\beta_u \\
&+ 2NpSI_c\delta\theta\omega(A)\alpha_0\beta_1\beta_u + 2NRSI_c\delta\theta\omega(A)\alpha_0\beta_1\beta_u \\
&- 2NpRSI_c\delta\theta\omega(A)\alpha_0\beta_1\beta_u + 2NI_uSI_c\delta\theta\omega(A)\alpha_0\beta_1\beta_u \\
&+ NI_uSI_c^2\gamma^2\delta\omega(A)\beta_1^2\beta_u - Np\delta I_uSI_c^2\gamma\theta\omega(A)\beta_1^2\beta_u \\
&+ NI_uSI_c^2\gamma^2\theta\omega(A)\beta_1^2\beta_u - NSI_c^2\gamma\delta\theta\omega(A)\beta_1^2\beta_u \\
&+ NpSI_c^2\gamma\delta\theta\omega(A)\beta_1^2\beta_u + NRSI_c^2\gamma\delta\theta\omega(A)\beta_1^2\beta_u \\
&- NpRSI_c^2\gamma\delta\theta\omega(A)\beta_1^2\beta_u + NI_uSI_c^2\delta\theta^2\omega(A)\beta_1^2\beta_u \\
&- NpRSI_c^2\delta\theta^2\omega(A)\beta_1^2\beta_u + NI_uSI_c^2\gamma^2\omega(A)\alpha_0\beta_1^2\beta_u \\
&- NSI_c^2\gamma\delta\omega(A)\alpha_0\beta_1^2\beta_u + NpSI_c^2\gamma\delta\omega(A)\alpha_0\beta_1^2\beta_u \\
&+ 2NI_uSI_c^2\gamma\delta\omega(A)\alpha_0\beta_1^2\beta_u + 2NI_uSI_c^2\gamma\theta\omega(A)\alpha_0\beta_1^2\beta_u \\
&- NSI_c^2\delta\theta\omega(A)\alpha_0\beta_1^2\beta_u + NpSI_c^2\delta\theta\omega(A)\alpha_0\beta_1^2\beta_u \\
&+ NRSI_c^2\delta\theta\omega(A)\alpha_0\beta_1^2\beta_u - NpRSI_c^2\delta\theta\omega(A)\alpha_0\beta_1^2\beta_u \\
&+ NI_uSI_c^2\delta\theta\omega(A)\alpha_0\beta_1^2\beta_u \Bigg) \Bigg/ N^2\left(1 + I_c\beta_1\right)^2.
\end{aligned}
$$

$$a_5 = \Big(- N^2 p\delta x\gamma^2\theta - N^2 RS\gamma^2\delta\theta + N^2 pRS\gamma^2\delta\theta + N^2 p\delta RS\gamma\theta^2$$
$$- N^2 R\gamma^2\delta\theta^2 + N^2\gamma^2\delta\theta\alpha_0 - N^2 R\gamma^2\delta\theta\alpha_0 - 2N^2 R\gamma\delta\theta^2\alpha_0$$
$$+ N^2 p\delta S\gamma\theta\alpha_1 - N^2 p\delta RS\gamma\theta\alpha_1 - N^2 p\delta RS\theta^2\alpha_1 - 2N^2 p\delta SI_c\gamma^2\theta\beta_1 -$$
$$2N^2 RSI_c\gamma^2 d\theta\beta_1 + 2N^2 pRSI_c\gamma^2 d\theta\beta_1 + 2N^2 p\delta RSI_u\gamma\theta^2\beta_1 +$$
$$2N^2 RI_c\gamma^2 d\theta^2\beta_1 - 2N^2 I_c\gamma^2\delta\theta\alpha_0\beta_1 - 2N^2 RI_c\gamma^2\delta\theta\alpha_0\beta_1 -$$
$$4N^2 RI_c\gamma\delta\theta^2\alpha_0\beta_1 - N^2 p\delta SI_c^2\gamma^2\theta\beta_1 - N^2 RSI_c^2\gamma^2\delta\theta\beta_1 + N^2 pRSI_c^2\gamma^2\delta\theta\beta_1^2$$
$$+ N^2 p\delta RSI_c^2\gamma\theta^2\beta_1^2 - N^2 RI_c^2\gamma^2\delta\theta^2\beta_1^2 + N^2 I_c^2\gamma^2\delta\theta\alpha_0\beta_1^2 - N^2 RI_c^2\gamma^2\delta\theta\alpha_0\beta_1^2 -$$
$$N^2 RI_c^2\gamma^2\theta^2\alpha_0\beta_1 - 2N^2 RI_c^2\gamma\delta\theta^2\alpha_0\beta_1 - Np\delta SI_c\gamma^2\theta\omega(A)\beta_c$$
$$+ NpSI_c\gamma^2\delta\theta\omega(A)\beta_c + Np\delta RS\gamma\theta^2\omega\beta_c + NSI_c\gamma^2\delta\omega(A)\alpha_0\beta_c -$$
$$Np\delta S\gamma\theta\omega(A)\alpha_0\beta_c + Np\delta RS\gamma\theta\omega(A)\alpha_0\beta_c - Np\delta SI_c\gamma\theta\omega(A)\alpha_0\beta_c$$
$$+ NSI_c\gamma^2\theta\omega(A)\alpha_0\beta_c + NSI_c\gamma d\theta\omega(A)\alpha_0\beta_c + NpSI_c\gamma\delta\theta\omega(A)\alpha_0\beta_c -$$
$$2Np\delta SI_c^2\gamma^2\theta\omega(A)\beta_1 + N^2 p\delta RSI_c\gamma^2\theta^2\beta_1\beta_c + 2Np\delta RSI_c\gamma^2\theta\omega(A)\beta_1\beta_c -$$
$$2Np\delta SI_c\gamma\theta\omega(A)\alpha_0\beta_1\beta_c + 2Np\delta RSI_c\gamma^2\theta\omega(A)\alpha_0\beta_1\beta_c -$$
$$2Np\delta SI_c^2\gamma^2\theta\omega(A)\alpha_0\beta_1 - Np\delta SI_c^3\gamma^2\theta\omega(A)\beta_1\beta_c + NpSI_c^3\gamma^2 d\theta\omega(A)\beta_c +$$
$$Np\delta SI_c^2\gamma^2\delta\theta\omega(A)\alpha_0\beta_1 + 2Np\delta RSI_c^2\gamma^2\theta\omega(A)\alpha_0\beta_1 - N^2 p\delta SI_c^3\gamma^2\theta\omega(A)\beta_1 -$$
$$Np\delta SI_c^2\gamma^2\delta\theta\omega(A)\beta_1\beta_c + N^2 pSI_c^2\gamma^2\delta\theta\omega(A)\beta_1\beta_c +$$
$$NSI_c^3\gamma^2\delta\omega(A)\alpha_0\beta_1^2\beta_c - Np\delta SI_c^2\gamma\theta\omega(A)\alpha_0\beta_1^2\beta_e +$$
$$NpRSI_c^2\gamma\theta\omega(A)\alpha_0\beta_1^2\beta_c - NpSI_c^3\gamma\theta\omega(A)\alpha_0\beta_1^2\beta_c +$$
$$NSI_c^3\gamma^2\theta\omega(A)\alpha_0\beta_1^2\beta_c + NSI_c^3\gamma\delta\theta\omega(A)\alpha_0\beta_1^2\beta_c +$$
$$NpSI_c^3\gamma\delta\theta\omega(A)\alpha_0\beta_1^2\beta_c + Np\delta RSI_c^2\theta^2\omega(A)\alpha_0\beta_1^2\beta_c -$$
$$NpI_uS\gamma^2\theta\omega(A)\beta_u + NpI_uS\gamma^2\delta\theta\omega(A)\beta_u + NRS\gamma\delta\theta^2\omega(A)\beta_u -$$
$$NpRS\gamma\delta\theta^2\omega(A)\beta_u + NI_uS\gamma^2\delta\omega(A)\alpha_0\beta_u - Np\delta I_uS\gamma\theta\omega(A)\alpha_0\beta_u +$$
$$NI_uS\gamma^2\theta\omega(A)\alpha_0\beta_u - NS\gamma\delta\theta\omega(A)\alpha_0\beta_u + NpS\gamma\delta\theta\omega(A)\alpha_0\beta_u +$$
$$NRS\gamma\delta\theta\omega(A)\alpha_0\beta_u - NpRS\gamma\delta\theta\omega(A)\alpha_0\beta_u + NI_uS\gamma\delta\theta\omega(A)\alpha_0\beta_u +$$
$$NpI_uS\gamma\delta\theta\omega(A)\alpha_0\beta_u + NRS\delta\theta^2\omega(A)\alpha_0\beta_u - NpRS\delta\theta^2\omega(A)\alpha_0\beta_u -$$
$$2NpI_uSI_c\gamma^2\theta\omega(A)\beta_1\beta_u + 2NpI_uSI_c\gamma^2\delta\theta\omega(A)\beta_1\beta_u +$$
$$2NRSI_c\gamma\delta\theta^2\omega(A)\beta_1\beta_u - 2NpRSI_c\gamma\delta\theta^2\omega(A)\beta_1\beta_u +$$
$$2NI_uSI_c\gamma^2\delta\omega(A)\alpha_0\beta_1\beta_u - 2NpI_uSI_c\gamma\theta\omega(A)\alpha_0\beta_1\beta_u +$$
$$2NI_uSI_c\gamma^2\theta\omega(A)\alpha_0\beta_1\beta_u - 2NSI_c\gamma\delta\theta\omega(A)\alpha_0\beta_1\beta_u +$$
$$2NpSI_c\gamma\delta\theta\omega(A)\alpha_0\beta_1\beta_u + 2NRSI_c\gamma\delta\theta\omega(A)\alpha_0\beta_1\beta_u -$$
$$2NpRSI_c\gamma\delta\theta\omega(A)\alpha_0\beta_1\beta_u + 2NI_uSI_c\gamma\delta\theta\omega(A)\alpha_0\beta_1\beta_u +$$
$$2NpI_uSI_c\gamma\delta\theta\omega(A)\alpha_0\beta_1\beta_u + 2NRSI_c\delta\theta^2\omega(A)\alpha_0\beta_1\beta_u -$$
$$2NpRSI_c\delta\theta^2\omega(A)\alpha_0\beta_1\beta_u\Big)/N^2\left(1 + I_c\beta_1\right)^2$$

$$
\begin{aligned}
a_6 = \Big(& N^2 p\delta RS\gamma^2\theta^2 - N^2 R\gamma^2\delta\theta^2\alpha_0 - N^2 p\delta RS\gamma\theta^2\alpha_1 + 2N^2 p\delta RSI_c\gamma^2\theta^2\beta_1 - \\
& 2N^2 RI_c\gamma^2\delta\theta^2\alpha_0\beta_1 + N^2 p\delta RSI_c^2\gamma^2\theta^2\beta_1^2 - N^2 RI_c^2\gamma^2\delta\theta^2\alpha_0\beta_1^2 - \\
& Np\delta SI_c\gamma^2\theta\omega(A)\alpha_0\beta_c + NpSI_c\gamma^2\delta\theta\omega(A)\alpha_0\beta_c + Np\delta RS\gamma\theta^2\omega(A)\alpha_0\beta_c - \\
& 2Np\delta SI_c^2\gamma^2\theta\omega(A)\alpha_0\beta_1\beta_c + 2NpSI_c^2\gamma^2\delta\theta\omega(A)\alpha_0\beta_1\beta_c + \\
& 2Np\delta RSI_c\gamma\delta\theta^2\omega(A)\alpha_0\beta_1\beta_c - Np_\delta SI_c^3\gamma^2\theta\omega(A)\alpha_0\beta_1^2\beta_c + \\
& NpSI_c^3\gamma^2\delta\theta\omega(A)\alpha_0\beta_1^2\beta_c + Np\delta RSI_c^2\gamma\theta^2\omega(A)\alpha_0\beta_1^2\beta_c - \\
& Np\delta I_u S\gamma^2\theta\omega(A)\alpha_0\beta_u + NpI_u S\gamma^2\delta\theta\omega(A)\alpha_0\beta_u + NRS\gamma\delta\theta^2\omega(A)\alpha_0\beta_u - \\
& NpRS\gamma\delta\theta^2\omega(A)\alpha_0\beta_u - 2Np\delta I_u SI_c\gamma^2\theta\omega(A)\alpha_0\beta_1\beta_u + \\
& 2NpI_u SI_c\gamma^2\delta\theta\omega(A)\alpha_0\beta_1\beta_u + 2NRSI_c\gamma\delta\theta^2\omega(A)\alpha_0\beta_1\beta_u - \\
& 2NpRSI_c\gamma\delta\theta^2\omega(A)\alpha_0\beta_1\beta_u - Np\delta I_u SI_c^2\gamma^2\theta\omega(A)\alpha_0\beta_1^2\beta_u + \\
& NpI_u SI_c^2\gamma^2\delta\theta\omega(A)\alpha_0\beta_1^{(2)}\beta_u + NRSI_c^2\gamma\delta\theta^2\omega(A)\alpha_0\beta_1^2\beta_u - \\
& NpRSI_c^2\gamma\delta\theta^2\omega(A)\alpha_0\beta_1^2\beta_u \Big) / N^2 \left(1 + I_c\beta_1\right)^2 .
\end{aligned}
$$

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