TMJM THE MINDANAWAN JOURNAL OF MATHEMATICS
OFFICIAL JOURNAL OF THE MINDANAO STATE UNIVERSITY - ILIGAN INSTITUTE OF TECHNOLOGY

Vol. 7 (2025), no. 1, pp. 87–107 ISSN: 2094-7380 (Print) | 2783-0136 (Online)
http://doi.org/10.62071/tmjm.v7i1.774

Bayesian Modeling of Zero-Inflated Count Time Series Using Adaptive MCMC on Dengue Incidence of Iligan and Tandag Cities, Philippines

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Received: 10th April 2025 Revised: 1st July 2025

Abstract

This study presents a Bayesian approach to modeling dengue incidence in Iligan and Tandag cities, Philippines, using integer-valued time series models. Recognizing the challenges posed by overdispersion, serial dependence, and excess zeros in dengue count data, we compare five probabilistic models: Generalized Poisson (GP), Log-Generalized Poisson (Log-GP), Negative Binomial (NB), Zero-Inflated Generalized Poisson (ZIGP), and Zero-Inflated Negative Binomial (ZINB) INGARCHX models. These models incorporate rainfall and temperature as lagged exogenous covariates. Parameter estimation is carried out using Adaptive Markov Chain Monte Carlo (MCMC) methods, and model performance is assessed via the Deviance Information Criterion (DIC) and residual diagnostics. Results reveal that the ZINB-INGARCHX model is best suited for the zero-inflated Tandag dataset, while the ZIGP-INGARCHX model provides the best fit for the overdispersed Iligan data. Findings highlight the importance of flexible count models and lagged environmental drivers in accurately capturing the dynamics of dengue transmission.

1 Introduction

Dengue fever, a mosquito-borne viral infection, poses a significant public health challenge in tropical and subtropical regions worldwide. Transmitted primarily by *Aedes aegypti* mosquitoes, the dengue virus causes symptoms ranging from mild fever to severe hemorrhagic conditions. The proliferation of dengue is closely linked to environmental factors that influence mosquito breeding and survival, such as temperature and rainfall [12].

The Philippines has experienced a notable increase in dengue cases in recent years. As of February 15, 2025, the Department of Health (DOH) reported 43,732 cases, marking a 56% rise

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2020 Mathematics Subject Classification: 62F15, 62M10, 62P10
Keywords and Phrases: Zero Inflation, Overdispersion, INGARCHX, Adaptive MCMC, Dengue Incidence
This research is supported by the DOST-ASTHRDP Scholarship Grant

compared to the 27,995 cases recorded during the same period in the previous year [9]. This upward trend underscores the urgent need for effective surveillance and intervention strategies to mitigate the disease's impact.

In Iligan City, located in Lanao del Norte, the DOH recorded 758 dengue cases in 2024, highlighting the city's vulnerability to outbreaks [8]. Similarly, Tandag City in Surigao del Sur reported 233 cases in the same year, reflecting a significant disease burden [10]. These figures emphasize the necessity for localized studies to understand and address the specific factors contributing to dengue transmission in these areas.

Accurate modeling of dengue incidence is crucial for predicting outbreaks and implementing timely interventions. Traditional models, such as the Autoregressive Integrated Moving Average (ARIMA), have been used to forecast dengue cases based on climatic variables [1]. However, dengue time series data often exhibit distinct features such as overdispersion, serial dependence, and an excess of zero counts, which challenge the assumptions of standard statistical methods.

To address these issues, researchers have proposed the use of zero-inflated and overdispersed count models. For example, Bayesian zero-inflated spatio-temporal models have been applied to dengue data in the Caraga region, successfully capturing both meteorological influences and spatial dependencies [17].

In parallel, hybrid machine learning approaches have also shown potential in handling zero-inflated structures and enhancing prediction accuracy [20]. Recent studies continue to highlight the value of Bayesian frameworks for dengue forecasting, including interdisciplinary spatiotemporal models that integrate lagged climatic covariates and use MCMC-based inference [2].

The methodological foundation of this study is motivated by the works of [16, 6]. These studies developed integer-valued transfer function models that address the unique features of count data, such as serial dependence, overdispersion, and zero-inflation. They also incorporate the influence of exogenous covariates with possible lagged effects.Their frameworks emphasized multiple transfer function models (MTFM) and demonstrated strong empirical performance in the Philippine context. In particular, it was found that rainfall and temperature had lagged effects of one and three weeks, respectively, and that zero-inflated models such as ZIGP MTFM and ZINB MTFM outperformed simpler alternatives in terms of predictive accuracy.

Motivated by these findings, this study applies and compares five probabilistic time series models—Generalized Poisson (GP) INGARCHX, Log-Generalized Poisson (Log-GP) INGARCHX, Negative Binomial (NB) INGARCHX, Zero-Inflated Generalized Poisson (ZIGP) INGARCHX, and Zero-Inflated Negative Binomial (ZINB) INGARCHX. These models are designed to accommodate serial dependence, overdispersion, and excess zeros, and incorporate lagged effects of environmental variables such as rainfall and temperature. By employing Adaptive Markov Chain Monte Carlo (MCMC) estimation, the study aims to identify robust modeling techniques for forecasting dengue incidence in Iligan City and Tandag City, ultimately contributing to more effective public health responses.

This study provides the first direct comparison of these five INGARCHX variants within a Philippine regional health context, demonstrating their ability to capture local dengue dynamics. In addition, the adaptive MCMC procedure serves not only to estimate model parameters but also to identify optimal lag structures for rainfall and temperature, offering practical value for data-driven disease forecasting.

2 Methodology

This study builds upon the modeling philosophy of [16, 6] by extending their framework to focus on core count-based INGARCHX models. While their original approach emphasized transfer

function dynamics, we implement a unified structure based on zero-inflated autoregressive count time series models that directly incorporate lagged exogenous effects.

In particular, we construct and analyze five variants within the INGARCHX framework: Generalized Poisson (GP), Log-Generalized Poisson (Log-GP), Negative Binomial (NB), Zero-Inflated Generalized Poisson (ZIGP), and Zero-Inflated Negative Binomial (ZINB). These models capture key features such as overdispersion, excess zeros, and temporal dependence while allowing for environmental covariates like rainfall and temperature to influence the conditional mean through lagged effects.

All models are estimated using an adaptive Markov Chain Monte Carlo (MCMC) algorithm that follows a two-phase strategy, combining Random Walk Metropolis-Hastings and Independent Kernel Metropolis-Hastings schemes to enhance convergence and sampling efficiency. This Bayesian framework allows us to perform full posterior inference and model comparison using the Deviance Information Criterion (DIC), thereby identifying the most appropriate model for each city.

This adaptive MCMC approach aligns with recent advances in Bayesian climate-based dengue forecasting that use MCMC-driven estimation for spatiotemporal models with lagged rainfall and temperature effects [2]. Similarly, recent studies such as the ARBOALVO framework apply Bayesian MCMC methods to handle spatial structure and environmental drivers in dengue time series [14], demonstrating the ongoing relevance of flexible MCMC estimation for vector-borne disease forecasting.

2.1 General Model Structure

Let $\{Y_t\}$ denote the weekly dengue case counts, and let $\mathcal{F}_{t-1}$ be the information set available up to time $t-1$, that is, $\mathcal{F}_{t-1} = \{Y_t, Y_{t-1}, \ldots; X_t, X_{t-1}, \ldots\}$. The general form of a zero-inflated INGARCHX model is given by:

$$Y_t \mid \mathcal{F}_{t-1} \sim \rho \cdot \delta_0 + (1-\rho) \cdot f(\lambda_t), \tag{1}$$

where δ_0 is a degenerate distribution at zero, $\rho \in [0, 1)$ is the zero-inflation parameter, and $f(\lambda_t)$ is a count distribution (e.g., generalized Poisson or negative binomial) with time-varying mean λ_t. The evolution of the conditional mean λ_t follows an INGARCHX(1,1) form:

$$\lambda_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_1 \lambda_{t-1} + \omega_1 X_{1,t-b_1} + \omega_2 X_{2,t-b_2}, \tag{2}$$

where $X_{1,t}$ and $X_{2,t}$ represent rainfall and temperature, respectively, and b_1, b_2 are their corresponding lag (delay) parameters. The parameters $\alpha_0 > 0$, $\alpha_1, \beta_1 \geq 0$, and $\alpha_1 + \beta_1 < 1$ are constrained to ensure stationarity. When the zero-inflation parameter $\rho = 0$, the general model reduces to its standard INGARCHX form with either a Poisson or Negative Binomial innovation.

Figure 1 illustrates the structure of the Zero-Inflated INGARCHX(1,1) model, showing how past counts, lagged climate covariates, and the zero-inflation mechanism interact to generate the predicted dengue counts.

http://doi.org/10.62071/tmjm.v7i1.774

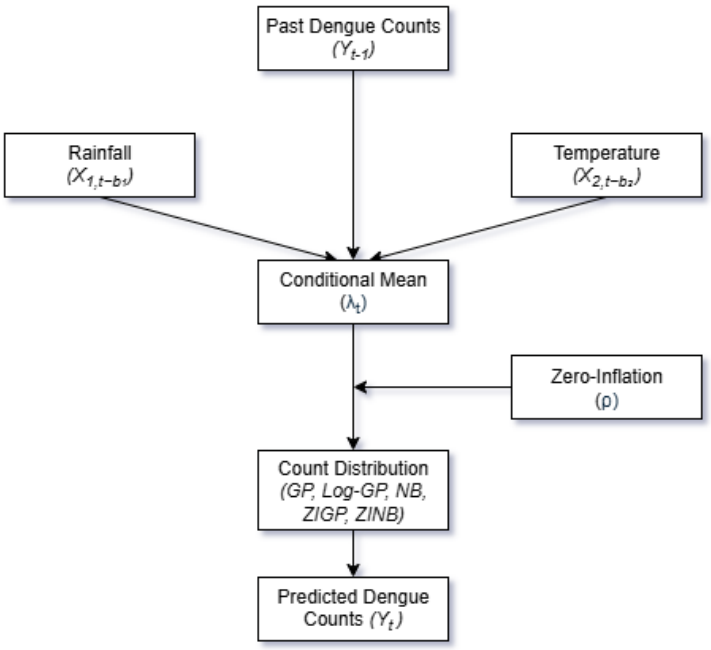

Figure 1: Schematic diagram of the Zero-Inflated INGARCHX(1,1) Model.

As shown in Figure 1, the Zero-Inflated INGARCHX(1,1) framework combines historical dengue incidence, lagged rainfall and temperature, and an autoregressive structure to calculate the conditional mean. This conditional mean then determines the parameters of the chosen count distribution while the zero-inflation component ρ adjusts for excess zeros in the data. This structure ensures that both temporal dependence and climate-driven variability are incorporated when modeling weekly dengue counts, addressing key features such as overdispersion and the prevalence of zero-case weeks, especially in low-incidence periods.

2.2 Specific Models

Zero-Inflated Negative Binomial INGARCHX (1,1)

This formulation is based on the studies of Chen, Liu, and Pingal [6], which introduced a zero-inflated negative binomial INGARCHX framework for modeling overdispersed and zero-inflated count data with exogenous covariates. In this model, the conditional distribution of Y_t is defined as:

$$Y_t \mid \mathcal{F}_{t-1} \sim \rho \cdot \delta_0 + (1-\rho) \cdot \mathrm{NB}(\lambda_t, r), \tag{3}$$

$$\lambda_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_1 \lambda_{t-1} + \omega_1 X_{1,t-b_1} + \omega_2 X_{2,t-b_2}. \tag{4}$$

Here, ρ is the zero-inflation probability, λ_t is the conditional mean of the negative binomial distribution at time t, and r is the dispersion (shape) parameter that captures the extra-Poisson variation typical of dengue case counts. The autoregressive structure, defined by α_0, α_1, and β_1, allows the model to capture the time-dependent nature of disease incidence, while satisfying the constraints $\alpha_0 > 0$ and $\alpha_1 + \beta_1 < 1$ to ensure positivity and stationarity of the process. The coefficients ω_1 and ω_2 quantify the influence of lagged rainfall and temperature ($X_{1,t}$ and $X_{2,t}$), with corresponding delays b_1 and b_2, allowing the model to reflect realistic incubation and transmission lags. When the zero-inflation parameter $\rho = 0$, this flexible structure reduces to the standard negative binomial INGARCHX model of Pingal and Chen [16].

Zero-Inflated Generalized Poisson INGARCHX (1,1)

This formulation is based on the study of [6], which proposed a zero-inflated generalized Poisson INGARCHX model to accommodate both overdispersion and zero-inflation in count time series.

The model assumes that, conditional on past values, Y_t follows:

$$Y_t \mid \mathcal{F}_{t-1} \sim \rho \cdot \delta_0 + (1-\rho) \cdot \mathrm{GP}(\kappa_t, \psi), \tag{5}$$

$$\kappa_t = \frac{1-\psi}{1-\rho}\left(\lambda_t + \omega_1 X_{1,t-b_1} + \omega_2 X_{2,t-b_2}\right), \tag{6}$$

$$\lambda_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_1 \lambda_{t-1}. \tag{7}$$

Here, ψ is the dispersion parameter of the generalized Poisson distribution. The remaining parameters retain the same interpretations as in the ZINB-INGARCHX model. Notably, when the zero-inflation parameter $\rho = 0$, the model reduces to the generalized Poisson INGARCHX model introduced by [16], capturing overdispersion without zero-inflation.

2.3 Model Assumptions

Assumptions in ZI-INGARCHX(1,1) models are crucial because they define the underlying data generation process and directly impact the model's ability to accurately capture the characteristics of the data. Meeting these assumptions ensures the model's validity, reliability, and interpretability. The following assumptions support all versions of the ZI–INGARCHX(1,1) models described above:

1. **Two-stage data-generation.** The weekly dengue counts Y_t follow a mixture model where it is conditional on the past information $\mathcal{F}_{t-1}$. In this setup, a structural zero generated with probability $\rho \in [0,1)$, and the count is drawn from a standard count distribution $f(\lambda_t)$ with time-varying mean λ_t. The zero-inflation parameter ρ captures the proportion of structual zero unexplained by the count model.

2. **Autoregressive dependence.** The conditional mean λ_t evolves over time according to the INGARCHX(1,1) specification:

$$\lambda_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_1 \lambda_{t-1} + \omega_1 X_{1,t-b_1} + \omega_2 X_{2,t-b_2},$$

   where $X_{1,t}$ and $X_{2,t}$ denote exogenous covariates (e.g., rainfall and temperature), and b_1, b_2 are their respective delays. The model captures dependence on past observations and past conditional means.

3. **Stationarity and ergodicity.** The parameters $\alpha_1, \beta_1 \geq 0$ are constrained such that $\alpha_1 + \beta_1 < 1$, ensuring that the process $\{\lambda_t\}$ is strictly stationary and ergodic. This guarantees that the process stabilizes over time, allowing valid inference and long-run behavior analysis.

4. **Exogenous covariates.** The inclusion of exogenous variables X_{1,b_1} and X_{2,b_2} transform the model into INGARCHX framework by allowing these covariates in the linear predictor λ_t. For the identity link function, there must be restriction $\boldsymbol{\omega}^\top \mathbf{X}_{t-b} > 0$ so that λ_t remains nonnegative. For λ_t with log-link, no sign restriction for $\boldsymbol{\omega}$ and $\mathbf{X}_{t-b}$.

5. **Properly specified maximum lags.** The maximum lag parameter is selected to ensure that the influence of exogenous covariates is temporally aligned with the dependent variable. The proper choice of lag is crucial for capturing the real-life delay between the changes in covariates and their impact on dengue incidence. Incorrect lag specification may lead to misleading conclusions about the covariate effects.

http://doi.org/10.62071/tmjm.v7i1.774

6. **Independence of errors.** Conditional on the past information $\mathcal{F}_{t-1}$, the innovations are assumed to be serially uncorrelated:

$$\varepsilon_t \perp \varepsilon_s \mid \mathcal{F}_{\min(t,s)-1}, \quad \text{for all } t \neq s.$$

This implies that the residuals should not exhibit temporal autocorrelation. Diagnostic tools such as autocorrelation plots, Ljung–Box tests, and posterior predictive checks can be used to verify this assumption. If violated, model respecification (e.g., increased lag order or improved covariate modeling) may be necessary.

2.4 Likelihood Function

Let $\boldsymbol{Y} = \{Y_1, Y_2, \ldots, Y_n\}$ denote the observed time series, and $\boldsymbol{X}$ the corresponding matrix of exogenous variables. Let $\boldsymbol{\theta}$ represent the complete set of model parameters. The likelihood function depends on the assumed distribution for the count component.

Likelihood Function for ZIGP-INGARCHX(1,1)

For the ZIGP-INGARCHX (1,1) model, where the count component follows a Generalized Poisson distribution with dispersion parameter ψ, the conditional likelihood function is given by:

$$\begin{aligned}\mathcal{L}_{\text{ZIGP}}(\boldsymbol{Y} \mid \boldsymbol{X}, \boldsymbol{\theta}) = &\prod_{Y_t=0} \{\rho + (1-\rho)\exp(-\kappa_t)\} \\ &\times \prod_{Y_t>0} \left\{(1-\rho) \cdot \frac{\kappa_t(\kappa_t + \psi Y_t)^{Y_t-1}}{Y_t!} \exp\left(-(\kappa_t + \psi Y_t)\right)\right\}, \end{aligned} \tag{8}$$

Likelihood Function for ZINB-INGARCHX (1,1)

For the ZINB-INGARCHX (1,1) model, where the count component follows a Negative Binomial distribution with mean λ_t and shape parameter $r > 0$, the conditional likelihood function is given by:

$$\begin{aligned}\mathcal{L}_{\text{ZINB}}(\boldsymbol{Y} \mid \boldsymbol{X}, \boldsymbol{\theta}) = &\prod_{Y_t=0} \left\{\rho + (1-\rho)\left(\frac{r}{r+\lambda_t}\right)^r\right\} \\ &\times \prod_{Y_t>0} \left\{(1-\rho) \cdot \binom{Y_t + r - 1}{Y_t} \left(\frac{r}{r+\lambda_t}\right)^r \left(\frac{\lambda_t}{r+\lambda_t}\right)^{Y_t}\right\}, \end{aligned} \tag{9}$$

These likelihoods accommodate both overdispersion and excess zeros, while dynamically evolving through lagged observations and exogenous meteorological influences.

2.5 Prior Specification

Under the Bayesian framework, prior distributions are specified for all model parameters to reflect prior beliefs and facilitate regularization. The following prior assumptions are adopted, following the specification structure in [16], who demonstrated their effectiveness in modeling zero-inflated count time series with environmental covariates:

– *Time series coefficients* $(\alpha_0, \alpha_1, \beta_1)$ are assigned a constrained uniform prior over the region $\{\alpha_0 > 0,\ \alpha_1, \beta_1 \geq 0,\ \alpha_1 + \beta_1 < 1\}$, ensuring positivity and stationarity of the conditional mean process.

– *Zero-inflation parameter* ρ and *dispersion parameter* ψ (in the generalized Poisson model) are given independent Uniform$(0, 1)$ priors, reflecting non-informative beliefs over their plausible ranges.

– *Shape parameter* r (in the ZINB model) and the regression coefficients (ω_1, ω_2), which quantify the effects of exogenous covariates such as rainfall and temperature, are each assigned Gamma priors with common shape and rate hyperparameters c_1 and c_2.

– *Delay parameters* b_1 and b_2, which determine the lag structure of the covariate effects, are modeled using discrete uniform distributions over the set $\{1, 2, 3\}$, allowing equal probability for each candidate lag length.

2.6 Bayesian Estimation and Computation

The posterior distribution of the model parameters $\boldsymbol{\theta}$ given the observed data $\boldsymbol{Y}$ and covariates $\boldsymbol{X}$ is obtained via Bayes' theorem:

$$p(\boldsymbol{\theta} \mid \boldsymbol{Y}, \boldsymbol{X}) \propto \mathcal{L}(\boldsymbol{Y} \mid \boldsymbol{X}, \boldsymbol{\theta}) \cdot p(\boldsymbol{\theta}), \tag{10}$$

where $\mathcal{L}(\boldsymbol{Y} \mid \boldsymbol{X}, \boldsymbol{\theta})$ is the likelihood function and $p(\boldsymbol{\theta})$ denotes the joint prior distribution of the parameters.

Posterior inference is carried out using Markov Chain Monte Carlo (MCMC) sampling. Given the non-standard forms of the full conditional distributions, we adopt a two-phase Adaptive MCMC procedure for efficient sampling. The first phase employs the Random-Walk Metropolis-Hastings (RWMH) algorithm to explore the posterior surface and adapt the proposal distribution based on empirical variance. The second phase uses an Independent Kernel Metropolis-Hastings (IKMH) scheme, which refines sampling by using a fixed proposal density centered on the approximated posterior mode. This strategy, originally proposed by Chen and So [3], is known for enhancing convergence, improving mixing, and reducing computational inefficiency in high-dimensional or non-standard models.

The strength of adaptive MCMC methods has been demonstrated in several Bayesian time series applications, including nonlinear INGARCHX models and count data modeling with threshold and regime-switching dynamics. In particular, [5] successfully applied adaptive MCMC to model negative binomial INGARCHX processes with covariates, while [4] extended this framework to Markov-switching structures for dengue incidence, showing high computational efficiency and reliable convergence diagnostics.

Each MCMC chain runs for $N = 20{,}000$ iterations, with the first $M = 8{,}000$ discarded as burn-in. The remaining $N - M = 12{,}000$ samples are used for posterior summaries and diagnostics, including trace plots, autocorrelation functions, Geweke's convergence test [11], and inefficiency factors [7].

Since the delay parameter b_k is discrete, its conditional posterior distribution follows a multinomial form with posterior probabilities given by:

$$p\left(b_k = j \mid \mathbf{Y}, \mathbf{X}, \boldsymbol{\theta}_{\setminus b_k}\right) = \frac{\mathcal{L}(\mathbf{Y} \mid \mathbf{X}, b_k = j, \boldsymbol{\theta}_{\setminus b_k})}{\sum_{i=1}^{b_0} \mathcal{L}(\mathbf{Y} \mid \mathbf{X}, b_k = i, \boldsymbol{\theta}_{\setminus b_k})}, \quad j = 1, \ldots, b_0.$$

This allows the algorithm to estimate optimal lag values for each exogenous variable in a data-driven manner within the Bayesian inference framework.

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2.7 Model Selection and Diagnostic Checking

Model comparison is conducted using the Deviance Information Criterion (DIC), introduced by Spiegelhalter et al. [18], which provides a Bayesian generalization of the Akaike Information Criterion (AIC). It is especially useful for hierarchical models and when posterior distributions are obtained via Markov Chain Monte Carlo (MCMC) methods.

The deviance is defined as:

$$D(\boldsymbol{\theta}) = -2\log \mathcal{L}(\mathbf{Y} \mid \boldsymbol{\theta}) + 2\log p(\mathbf{Y}), \tag{11}$$

where $\mathcal{L}(\mathbf{Y} \mid \boldsymbol{\theta})$ is the likelihood of the observed data given the parameters. The DIC is then computed as:

$$\text{DIC} = \overline{D(\boldsymbol{\theta})} + p_D, \tag{12}$$

where $\overline{D(\boldsymbol{\theta})}$ is the posterior mean of the deviance, and $p_D = \overline{D(\boldsymbol{\theta})} - D(\overline{\boldsymbol{\theta}})$ is the effective number of parameters, with $\overline{\boldsymbol{\theta}}$ being the posterior mean of the parameters. A model with the lowest DIC value is preferred, indicating a better balance between model fit and complexity.

Model adequacy is further evaluated using standardized Pearson residuals to detect potential misspecification. Convergence and sampling efficiency of the MCMC chains are assessed through the Geweke diagnostic and inefficiency factors. Additionally, visual inspection using trace plots and autocorrelation function (ACF) plots is conducted to assess mixing behavior and confirm convergence.

For the model identified as best based on the DIC, in-sample prediction is carried out to assess its ability to replicate the observed data. This allows further validation of the model's adequacy and practical relevance in capturing the underlying dynamics of dengue incidence.

We calculate the standardized residual proposed by [13], for diagnostic checking:

$$e_t = \frac{Y_t - E\left(Y_t \mid \mathcal{Y}_{t-1}, \mathcal{X}_{t-b}\right)}{\sqrt{\text{Var}\left(Y_t \mid \mathcal{Y}_{t-1}, \mathcal{X}_{t-b}\right)}}, \tag{13}$$

where $E\left(Y_t \mid \mathcal{Y}_{t-1}, \mathcal{X}_{t-b}\right)$ represents the conditional mean and $\text{Var}\left(Y_t \mid \mathcal{Y}_{t-1}, \mathcal{X}_{t-b}\right)$ represents the conditional variance of the model. The mean of e_t must be zero, and its variance should be one and not exhibit autocorrelation.

2.8 Data and Meteorological Variables

This study analyzes dengue hemorrhagic fever (DHF) incidence in Tandag City, Surigao del Sur, and Iligan City, Lanao del Norte — two urban centers in Mindanao, Philippines, that exhibit contrasting dengue transmission patterns. Weekly DHF case data were collected from regional health offices, covering multiple years to capture seasonal and interannual variation. To account for environmental drivers, the models also include weekly cumulative rainfall and average temperature as exogenous covariates, given their known influence on mosquito breeding conditions and dengue spread.

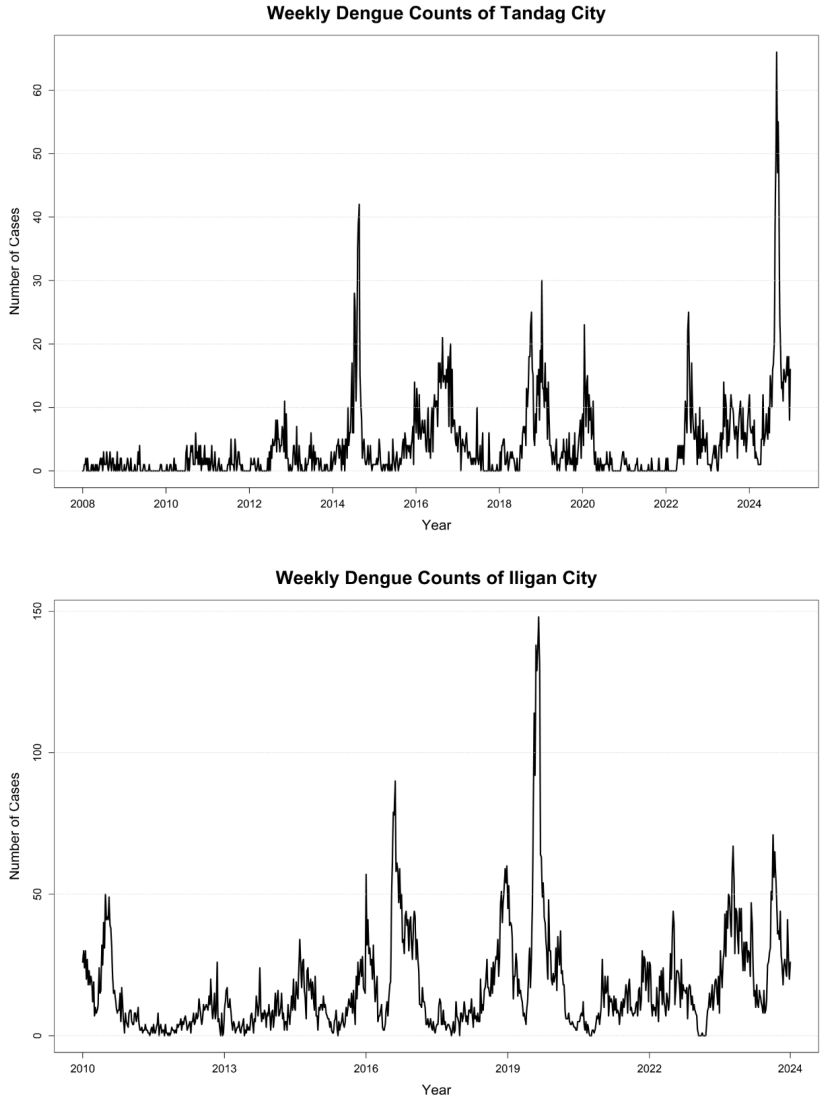

Figure 2: Weekly time series plot of dengue fever cases in Tandag City (2008–2024) and Iligan City (2010–2024).

The two datasets in Figure 2 were selected due to their contrasting distributional characteristics: Tandag City exhibits zero-inflation in reported dengue cases, while Iligan City shows evidence of zero-deflation. This contrast allows for a robust evaluation of the proposed model's ability to account for varying zero-count behaviors in disease incidence data.

Weekly dengue case counts were obtained from the Department of Health (DOH) in CARAGA Region and Region 10, Philippines. The Tandag City data spans from January 2008 to December 2024, while the Iligan City data covers January 2010 to December 2024.

Table 1 summarizes the descriptive statistics of the weekly dengue case counts in Tandag City and Iligan City. A key characteristic of the data in both locations is the presence of overdispersion, as evidenced by the fact that the sample variance substantially exceeds the mean in both cases—particularly in Iligan City, where the variance is nearly twenty times the mean. This property violates the equidispersion assumption of the Poisson model and thus motivates the use of more flexible count models such as the Negative Binomial and Generalized Poisson.

http://doi.org/10.62071/tmjm.v7i1.774

Table 1: Descriptive statistics of weekly dengue cases

City	Mean	Variance	Min	Q_1	Median	Q_3	Max	Zeros (%)
Tandag City	4.034	41.958	0	0	2	5	66	30.43
Iligan City	16.800	327.288	0	5	11	23	148	2.82

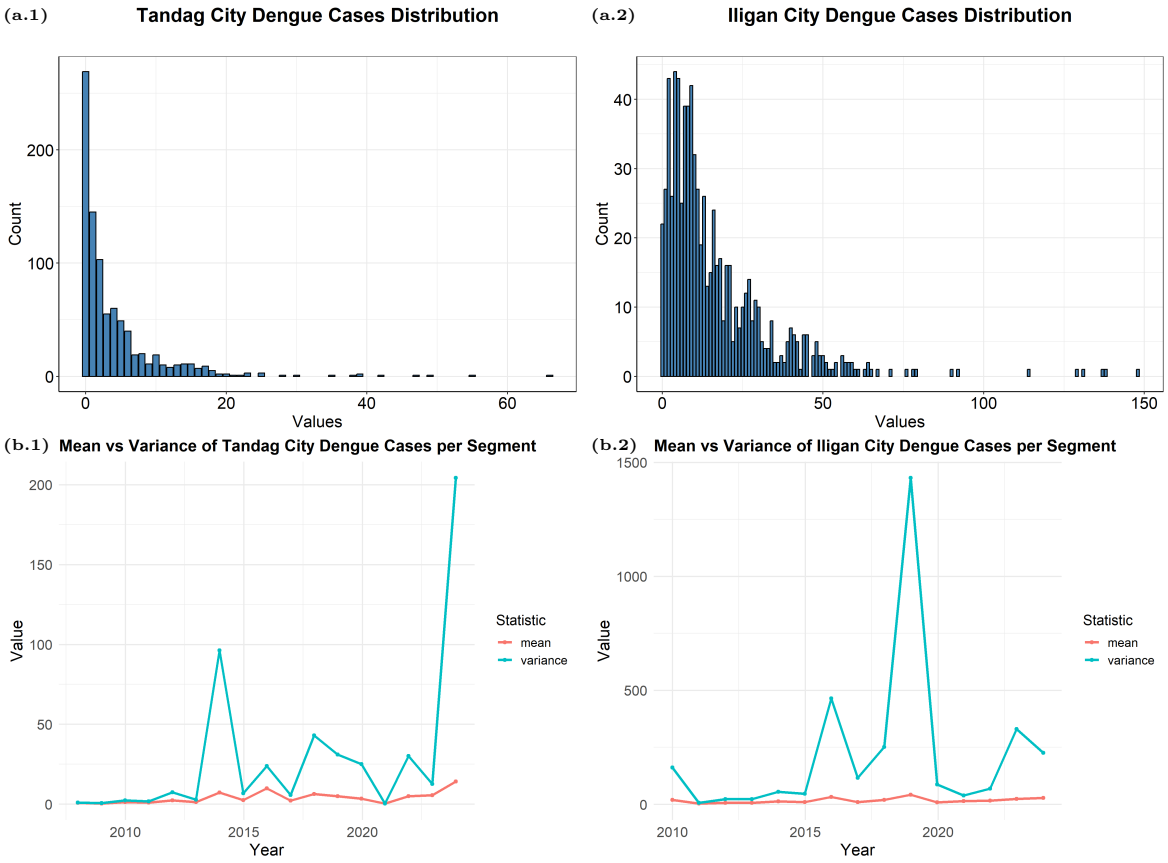

Figure 3: (a) Distribution of dengue cases in Tandag City and Iligan City, and (b) Segmented analysis highlights mean and variance of dengue cases per segment, emphasizing variability across different time periods.

Figure 3 complements these summary statistics by visualizing the distributional characteristics and temporal patterns of the two time series. Subplots (a.1) and (a.2) show the overall distribution of weekly dengue counts in Tandag and Iligan, respectively, highlighting the difference in zero counts and spread. The segmented plots (b.1) and (b.2) further illustrate how the mean and variance fluctuate over different periods, emphasizing local variability and the need to account for changing patterns in transmission. The high proportion of zeros in Tandag City (30.43%) is clearly visible, supporting a zero-inflated modeling approach to capture structural or excess zeros. In contrast, Iligan City has relatively few zero weeks (2.82%), although strong overdispersion remains a dominant feature. Together, these insights reinforce the importance of choosing model structures that reflect the contrasting transmission characteristics of the two locations.

To incorporate meteorological influences on dengue transmission, weekly cumulative rainfall and average temperature data were obtained from [15]. These climate variables have been widely recognized as significant environmental drivers of dengue outbreaks, as they directly affect the

lifecycle of the Aedes mosquito vector and the dynamics of virus transmission. Rainfall creates breeding habitats for mosquitoes through the accumulation of stagnant water, while temperature influences mosquito biting rates, development cycles, and viral replication within the vector [12]. As such, both rainfall and temperature have been extensively used as exogenous covariates in time series models of dengue incidence.

Table 2 presents the descriptive statistics of the weekly rainfall and temperature for the two study sites—Tandag City and Iligan City. These statistics provide an overview of the central tendency and range of the meteorological variables used in the analysis. To complement this, Figure 4 visualizes the temporal trends of these variables, highlighting seasonal patterns and variability across the two cities. The top panels depict the weekly rainfall and temperature for Tandag City (2008–2024), while the bottom panels correspond to Iligan City (2010–2024).

Table 2: Descriptive summary of meteorological variables

Location	**Rainfall (mm)**				**Temperature (°C)**			
	Median	**Mean**	**Min**	**Max**	**Median**	**Mean**	**Min**	**Max**
OpenWeather (Tandag)	49.89	65.20	0	770.02	26.43	26.38	24.12	28.45
OpenWeather (Iligan)	41.30	48.35	0.05	268.63	28.62	28.60	25.54	30.35

The meteorological profiles of Iligan City and Tandag City reveal both similarities and differences that affect dengue dynamics. While both cities experience seasonal rainfall and warm temperatures, Tandag shows higher rainfall variability and extreme weather events, whereas Iligan maintains consistently higher average temperatures. This variation supports the model's flexibility to account for diverse climatic drivers.

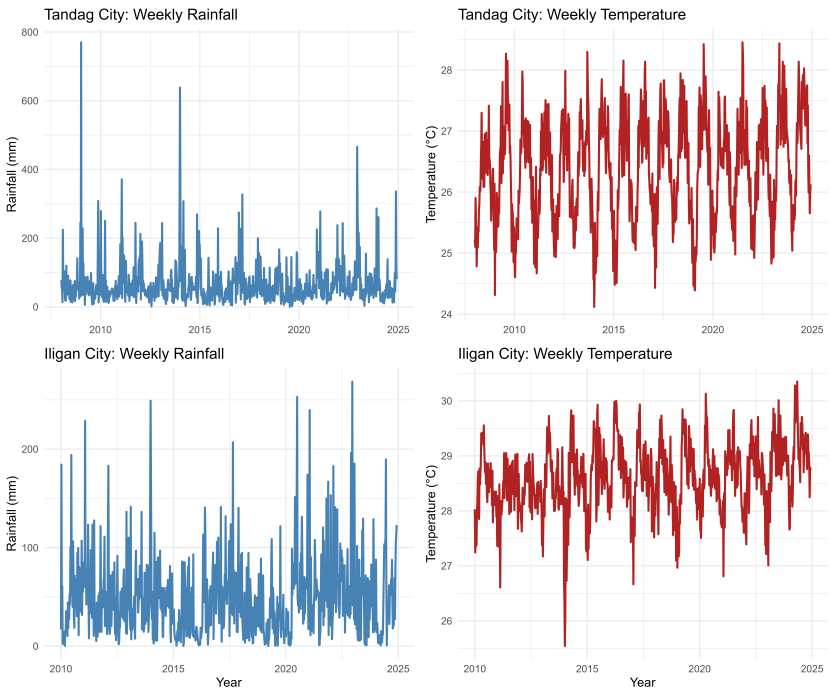

Figure 4: Weekly rainfall and temperature trends in Tandag City and Iligan City. The top row shows the rainfall and temperature in Tandag (2008–2024), while the bottom row displays the corresponding variables in Iligan (2010–2024).

http://doi.org/10.62071/tmjm.v7i1.774

As shown in both the Table 2 and the Figure 4, rainfall in Tandag exhibits higher variability and extreme values compared to Iligan. In contrast, temperature in Iligan remains consistently higher across the observed period. Given these disparities in scale and distribution, all covariates were standardized prior to modeling to ensure comparability and numerical stability during estimation.

The following transformation was used:

$$x^*_{i,t} = \frac{X_{i,t} - \min(X_{i,t})}{S_x^{(i)}}, \quad t = 1, \ldots, n; \quad i = 1, 2,$$

where $S_x^{(i)}$ denotes the standard deviation of the original covariate X_i. This scaled version $x^*_{i,t}$ ensures that rainfall and temperature are on a similar scale, aiding in efficient parameter estimation within the Bayesian framework.

3 Checking Assumptions

Before fitting the models, it is important to check if the data meet the assumptions. Checking these assumptions helps make sure the model accurately represents how the counts and zeros are produced, that the autoregressive terms properly capture patterns over time, and that the process is stable enough for reliable analysis and forecasting.

3.1 Two-stage data-generation

In the weekly dengue data from Tandag City and Iligan City, the zeros in the data appear to arise from both structural and sampling mechanisms. Structural zeros likely occur during weeks when environmental conditions, such as low rainfall or cooler temperatures, do not help in mosquito breeding and virus transmission or when there is public health interventions (e.g., fogging, community cleanup drives) effectively suppress outbreaks. On the other hand, sampling zeros may result from inherent randomness in case occurrence, particularly during transition weeks between low- and high-transmission seasons, or from underreporting due to delayed diagnostics or limited health facility access. Hence, the two-stage data-generation assumption is met.

3.2 Autoregressive dependence

The assumption of autoregressive dependence is satisfied in the weekly dengue case data for both Tandag City and Iligan City. As shown in the ACF plots in Figure 5, the lagged autocorrelation coefficients slowly decay and remain significantly outside the confidence limits 95%. This suggest temporal dependence in the data. This is further supported by the results of the Ljung-Box test in Table 3 where both cities produce extremely high chi-square statistics and p-values less than 2.2e–16, confirming statistically significant autocorrelation between multiple lags. These findings justify the use of an INGARCHX framework.

Table 3: Ljung-Box test for detecting autocorrelation in the weekly dengue time series data.

Location	χ^2	**df**	p**-value**
Tandag City	3033.1	10	<2.2e-16
Iligan City	3770.4	30	<2.2e-16

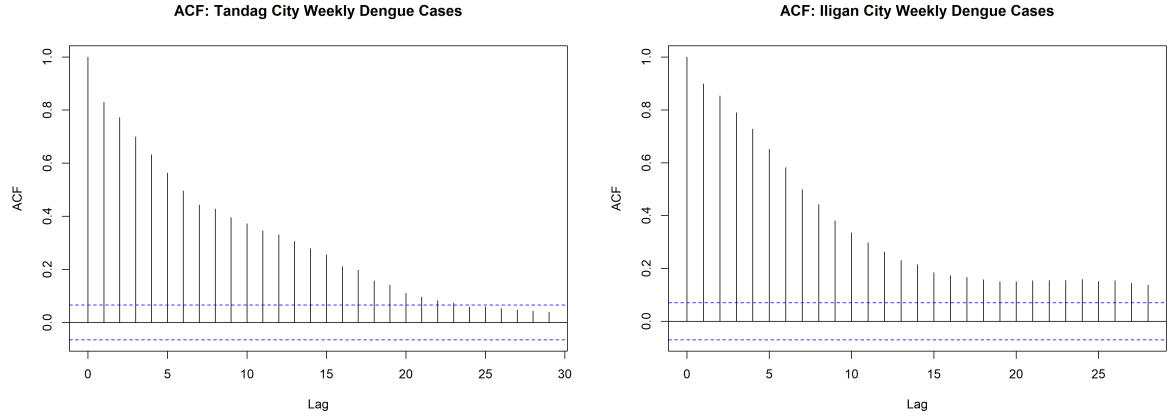

Figure 5: ACF functions for Tandag City and Iligan City dengue time series. The blue dashed lines mark approximate 95% significance bounds; bars outside these bounds indicate statistically significant lagged dependence.

3.3 Stationarity and ergodicity

The stationarity assumption is supported by the results of the Augmented Dickey–Fuller (ADF) test applied to the weekly dengue case series in both Tandag City and Iligan City presented in Table 4. The ADF test statistics are –5.3306 and –5.705, respectively, with both have p-values less than 0.01. These results provide strong evidence against the presence of a unit root, indicating that the time series are stationary. Stationarity is a key requirement for the validity of the INGARCHX model. This ensures that the temporal dynamics of the conditional mean process λ_t are stable over time and do not exhibit uncontrolled drift.

Table 4: Stationary test using Augmented Dickey-Fuller test for the weekly dengue time series data.

Location	**Statistic**	**Lag Order**	p**-value**
Tandag City	-5.3306	9	<0.01
Iligan City	-5.705	9	<0.01

3.4 Exogenous variables and maximum lag

In this study, rainfall and average temperature are incorporated as exogenous variables to account for environmental effects of dengue transmission. These meteorological factors are known to influence mosquito breeding and virus dynamics. To capture these delayed effects appropriately, a maximum lag of three weeks is imposed on both covariates. This choice is guided by the paper of [16], which identified a three-week lag as the optimal period reflecting the biological and ecological response time between weather conditions and reported dengue cases.

4 Results and Discussion

This section presents the empirical results from the Bayesian estimation of five count time series models applied to dengue incidence data in Tandag and Iligan cities. The models in-

http://doi.org/10.62071/tmjm.v7i1.774

clude the Generalized Poisson INGARCHX (GP-INGARCHX), Log-linear GP-INGARCHX (Log GP-INGARCHX), Negative Binomial INGARCHX (NB-INGARCHX), Zero-Inflated GP-INGARCHX (ZIGP-INGARCHX), and Zero-Inflated Negative Binomial INGARCHX (ZINB-INGARCHX). The primary evaluation criterion for model performance is the Deviance Information Criterion (DIC), where a lower DIC value indicates a better trade-off between model complexity and goodness-of-fit. Additionally, convergence diagnostics, parameter estimates, and model residuals are analyzed to assess model adequacy.

Table 5: Parameter Estimates and DIC by Model and City

City	Model	α_0	α_1	β	ρ	ω_1	ω_2	r	ψ	b_1	b_2	DIC
Tandag	GP-INGARCHX	0.0125	0.2953	0.4950	-	0.0997	0.0713	-	0.3447	1	1	3421.068
	Log GP-INGARCHX	0.0025	0.6308	0.1053	-	0.0168	0.0101	-	0.3702	3	1	3759.726
	NB-INGARCHX	0.0295	0.0226	0.2079	-	3.2843	7.5393	1.0013	-	1	3	5745.523
	ZIGP-INGARCHX	0.0339	0.4453	0.5054	0.0060	0.0759	0.0597	-	0.3436	1	1	3418.300
	ZINB-INGARCHX	0.0080	0.1363	0.5419	0.0078	0.0266	0.0300	2.9985	-	1	1	**3406.990**
Iligan	GP-INGARCHX	0.0422	0.3585	0.3282	-	0.1850	0.1281	-	0.4114	3	1	4784.618
	Log GP-INGARCHX	0.0053	0.7214	0.0970	-	0.0204	0.0091	-	0.4139	3	1	4903.175
	NB-INGARCHX	0.0959	0.0247	0.2311	-	4.5307	11.1470	1.0012	-	3	3	6759.287
	ZIGP-INGARCHX	0.1060	0.6046	0.3375	0.0026	0.1226	0.1184	-	0.4098	3	1	**4774.750**
	ZINB-INGARCHX	0.0128	0.0800	0.3867	0.0096	0.0154	0.0244	7.0924	-	1	1	4787.283

Table 5 shows the parameter estimates and DIC values for different models applied to the dengue incidence data in both cities. For Tandag, the **ZINB-INGARCHX** model achieved the lowest DIC (3406.99), suggesting it best captures the zero-inflation and overdispersion in the data. In Iligan, the **ZIGP-INGARCHX** model yielded the best fit with a DIC of 4774.750. Models that do not account for zero-inflation, such as GP and NB variants, yielded substantially higher DICs, highlighting the importance of zero-inflation mechanisms and distributional flexibility in modeling dengue time series data.

Table 6 presents the posterior estimates and convergence diagnostics for the Zero-Inflated INGARCHX(1,1) models fitted to the Tandag and Iligan dengue data. For Tandag's ZINB-INGARCHX model, the intercept ($\alpha_0 = 0.0080$) represents the baseline expected number of dengue cases when there is no past information and no climate effect. This very low value means that when conditions are neutral, the expected weekly count is near zero, which is realistic for Tandag. The parameter ($\alpha_1 = 0.1363$) implies that for every additional dengue case reported in the previous week, the expected number of cases this week increases by about 0.136. This shows that dengue cases in Tandag tend to cluster for short periods rather than appearing in isolation. Meanwhile, the past mean effect ($\beta = 0.5419$) means that what was expected last week still has a strong effect on what is expected this week. This indicates that dengue case numbers in Tandag generally continue over time rather than changing suddenly from week to week.

The dispersion parameter ($r = 2.9985$) confirms that the negative binomial distribution captures extra variation in the counts beyond what a simple Poisson process would predict. The zero-inflation parameter ($\rho = 0.0078$) is very low, meaning there is almost no extra probability of having additional zero-case weeks once the count model is in place. The rainfall effect ($\omega_1 = 0.0266$) means that for every unit increase in rainfall, the expected number of cases rises by about 0.027, while the temperature effect ($\omega_2 = 0.0300$) means that each unit increase in temperature raises the expected number by about 0.030. These effects confirm that rainfall and temperature help create conditions for mosquitoes to breed and transmit dengue. The identified lags ($b_1 = 1$, $b_2 = 1$) show that both rainfall and temperature influence dengue cases with about a one-week delay, matching known mosquito life cycles.

In Iligan's ZIGP-INGARCHX model, the intercept ($\alpha_0 = 0.1060$) shows a higher baseline

Table 6: MCMC Results and Diagnostics for the ZINB-INGARCHX (Tandag) and ZIGP-INGARCHX (Iligan) Models

Model	Param.	Mean	Med.	Std.	$\mathcal{P}_{2.5}$	$\mathcal{P}_{97.5}$	Z	IF
Tandag Data (ZINB-INGARCHX)								
	α_0	0.0080	0.0076	0.0048	0.0006	0.0183	-0.8109	4.64
	α_1	0.1363	0.1363	0.0098	0.1182	0.1561	0.9484	4.43
	β	0.5419	0.5424	0.0284	0.4858	0.5969	-1.5897	3.26
	r	2.9985	2.9977	0.0739	2.8544	3.1458	0.2266	3.62
	ρ	0.0078	0.0061	0.0067	0.0002	0.0253	0.7815	4.90
	ω_1	0.0266	0.0265	0.0079	0.0119	0.0423	1.8384	2.92
	ω_2	0.0300	0.0301	0.0041	0.0221	0.0382	0.7635	4.21
	b_1	1						
	b_2	1						
Iligan Data (ZIGP-INGARCHX)								
	α_0	0.1060	0.0968	0.0689	0.0059	0.2629	0.7837	4.32
	α_1	0.6046	0.6043	0.0238	0.5577	0.6518	-0.3492	2.69
	β	0.3375	0.3381	0.0257	0.2865	0.3880	0.9559	2.57
	ψ	0.4098	0.4098	0.0166	0.3771	0.4431	-1.4240	4.03
	ρ	0.0026	0.0019	0.0023	0.0001	0.0085	-1.1768	4.82
	ω_1	0.1226	0.1210	0.0416	0.0464	0.2095	-1.5496	2.87
	ω_2	0.1184	0.1187	0.0198	0.0799	0.1570	0.5625	3.76
	b_1	3						
	b_2	1						

risk than Tandag, meaning that even with no past cases or climate triggers, Iligan has a greater expected dengue count. The parameter ($\alpha_1 = 0.6046$) means that for every additional dengue case last week, the expected number of cases this week increases by about 0.605. This implies that dengue outbreaks in Iligan are more strongly clustered and can spread more quickly. The past mean effect ($\beta = 0.3375$) means that last week's expected cases still have a notable effect on this week's expected value, which helps smooth sudden spikes or drops.

The dispersion parameter ($\psi = 0.4098$) captures the extra variability typical of dengue counts under the generalized Poisson. The zero-inflation parameter ($\rho = 0.0026$) is low, showing that few extra zeros remain once the count model is included. For Iligan, rainfall ($\omega_1 = 0.1226$) and temperature ($\omega_2 = 0.1184$) both have larger positive effects, meaning that higher rainfall and temperatures strongly increase expected dengue cases. The lag for rainfall ($b_1 = 3$) means its impact appears after about three weeks, while the temperature effect ($b_2 = 1$) appears after one week, both consistent with mosquito breeding and virus incubation.

The inefficiency factors (IFs) for both cities are well below 5, showing that the adaptive MCMC sampling mixed well and produced reliable estimates. Notably, the IFs for rainfall and temperature are low for both Tandag ($\omega_1 = 2.92$, $\omega_2 = 4.21$) and Iligan ($\omega_1 = 2.87$, $\omega_2 = 3.76$), confirming stable and efficient posterior estimation.

These results show that past cases, expected trends, and climate conditions combine to predict weekly dengue counts, with realistic time lags and overdispersion handled by the chosen model. This aligns with known transmission patterns and shows that dengue case patterns tend to continue over time rather than change suddenly.

Finally, the identified lags—$b_1 = 1$ and $b_2 = 1$ for Tandag, and $b_1 = 3$, $b_2 = 1$ for Iligan—are consistent with known biological mechanisms, where rainfall creates mosquito breeding grounds within a week, and temperature influences mosquito-virus dynamics with slightly delayed effects.

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Together, these results highlight both the statistical adequacy and biological plausibility of the proposed models.

The performance of the Bayesian Zero-Inflated Negative Binomial INGARCHX (ZINB-INGARCHX) model is evaluated to assess its adequacy in modeling weekly dengue incidence in Tandag City, incorporating rainfall and temperature as exogenous predictors. As illustrated in Figure 6, the model closely captures the temporal dynamics of dengue outbreaks, with predicted values aligning well with observed case counts, including both outbreak peaks and periods of low transmission.

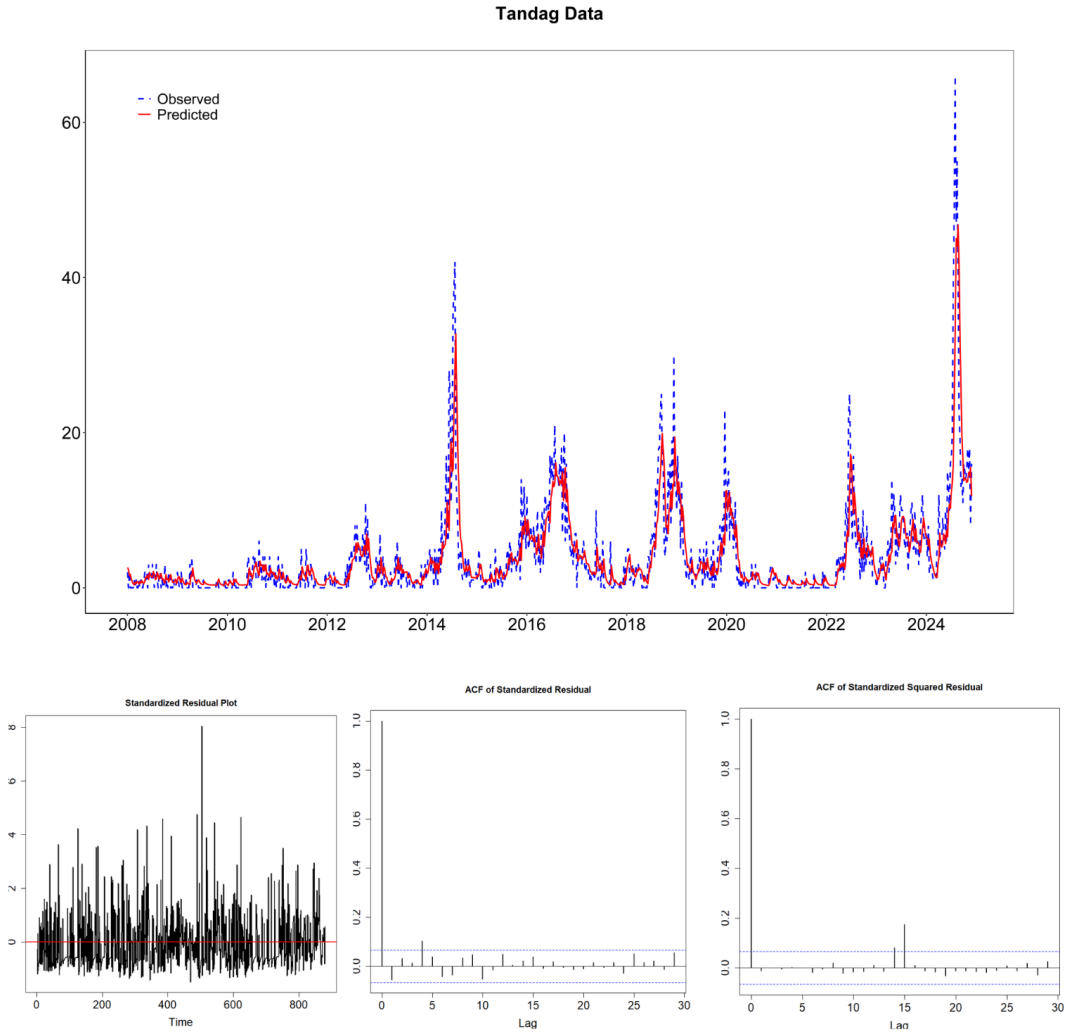

Figure 6: Model diagnostics for the ZINB-INGARCHX model applied to dengue incidence in Tandag City. Upper panel: in-sample prediction plot; Lower panel: standardized residuals and autocorrelation function (ACF) of the residuals.

The residual diagnostics shown in the lower panel of Figure 6 further validate the model's adequacy. The standardized residuals fluctuate randomly around zero, and the ACF bars fall within the 95% confidence bounds, indicating no significant autocorrelation. These results confirm that the ZINB-INGARCHX model effectively captures temporal dependence and randomness, supporting its suitability for surveillance and predictive applications.

A similar diagnostic evaluation is conducted for Iligan City using the Zero-Inflated Generalized Poisson INGARCHX (ZIGP-INGARCHX) model. Figure 7 summarizes its predictive

accuracy and residual behavior.

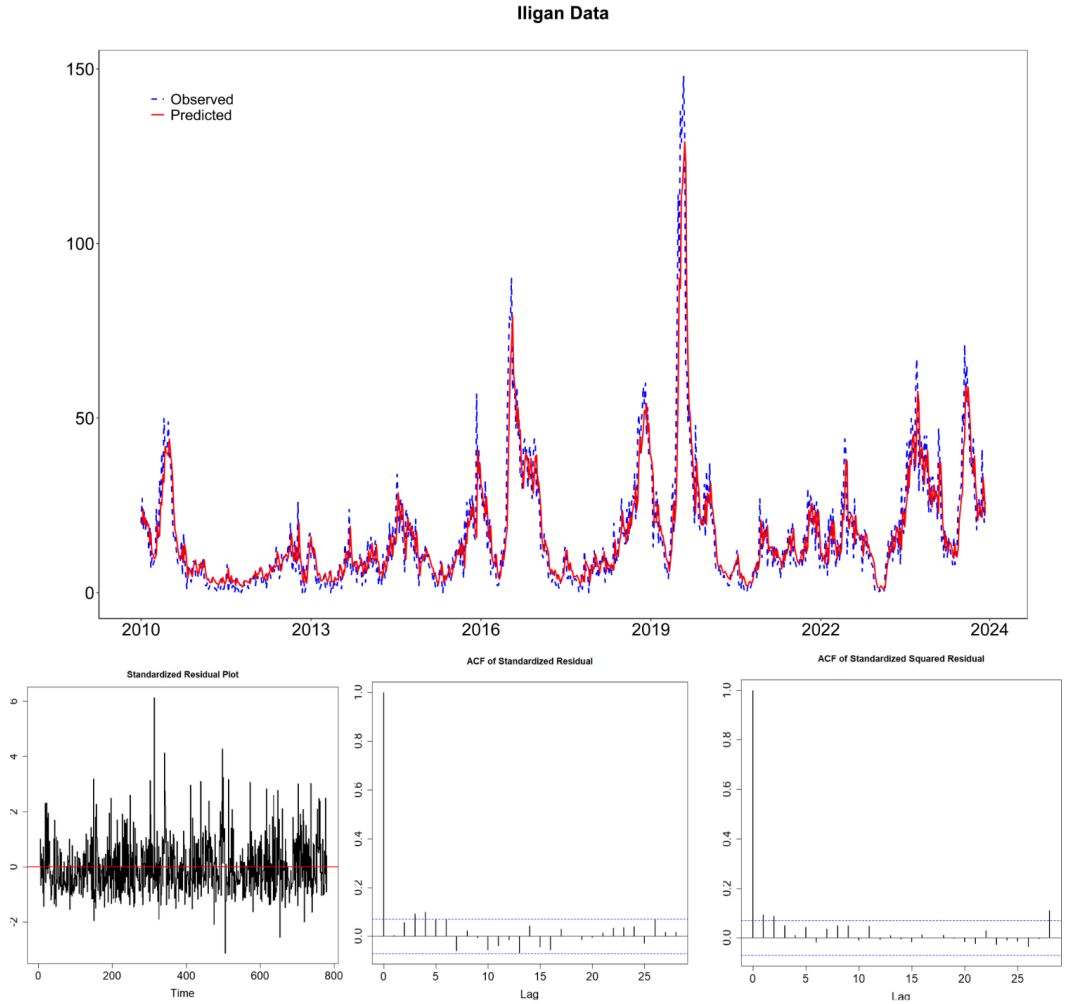

Figure 7: Model diagnostics for the ZIGP-INGARCHX model applied to dengue incidence in Iligan City. Upper panel: in-sample prediction plot; Lower panel: standardized residuals and autocorrelation function (ACF) of residuals.

The upper panel in Figure 7 indicates that the model successfully captures the observed weekly dengue cases, reflecting its responsiveness to underlying epidemic trends and environmental factors. Meanwhile, the residuals in the lower panel exhibit no discernible pattern and show no significant autocorrelation, as the ACF values remain within the 95% confidence bounds. These diagnostics affirm the reliability of the ZIGP-INGARCHX model in capturing the overdispersion and zero-inflation characteristics of the Iligan dataset, while accounting for delayed climatic influences.

Conclusion

This study confirms the effectiveness of zero-inflated Bayesian count time series models, particularly the INGARCHX framework, for capturing dengue dynamics in Iligan and Tandag cities. By integrating lagged rainfall and temperature, the models account for key environmental influences. Building on the frameworks of [16, 6], this approach addresses overdispersion, serial

http://doi.org/10.62071/tmjm.v7i1.774

dependence, and zero-inflation in local count data. Results show that the ZINB-INGARCHX model best fits Tandag's highly zero-inflated data, while the ZIGP-INGARCHX model suits Iligan's overdispersed but low-zero pattern. The adaptive MCMC estimation achieved good convergence and reliable posterior inference.

Beyond methodology, this study supports global sustainability goals [19], including **SDG #3** (health), **SDG #11** (resilient communities), and **SDG #13** (climate action) by modeling how climatic factors shape disease risk. The results emphasize the value of flexible count models and Bayesian approaches for location-specific dengue forecasting.

However, this work has limitations. Reported case and weather data may include measurement errors or underreporting. The models were validated only in-sample, which is appropriate for this initial methodological comparison focused on identifying the best-fitting structures and demonstrating their capacity to replicate observed local patterns. Nevertheless, other factors such as human movement or vector control were not included. To strengthen future work, real-time out-of-sample forecasting, integration of mobility and socio-economic data, and pilot testing as an early warning system with local health agencies are recommended.

5 Appendix

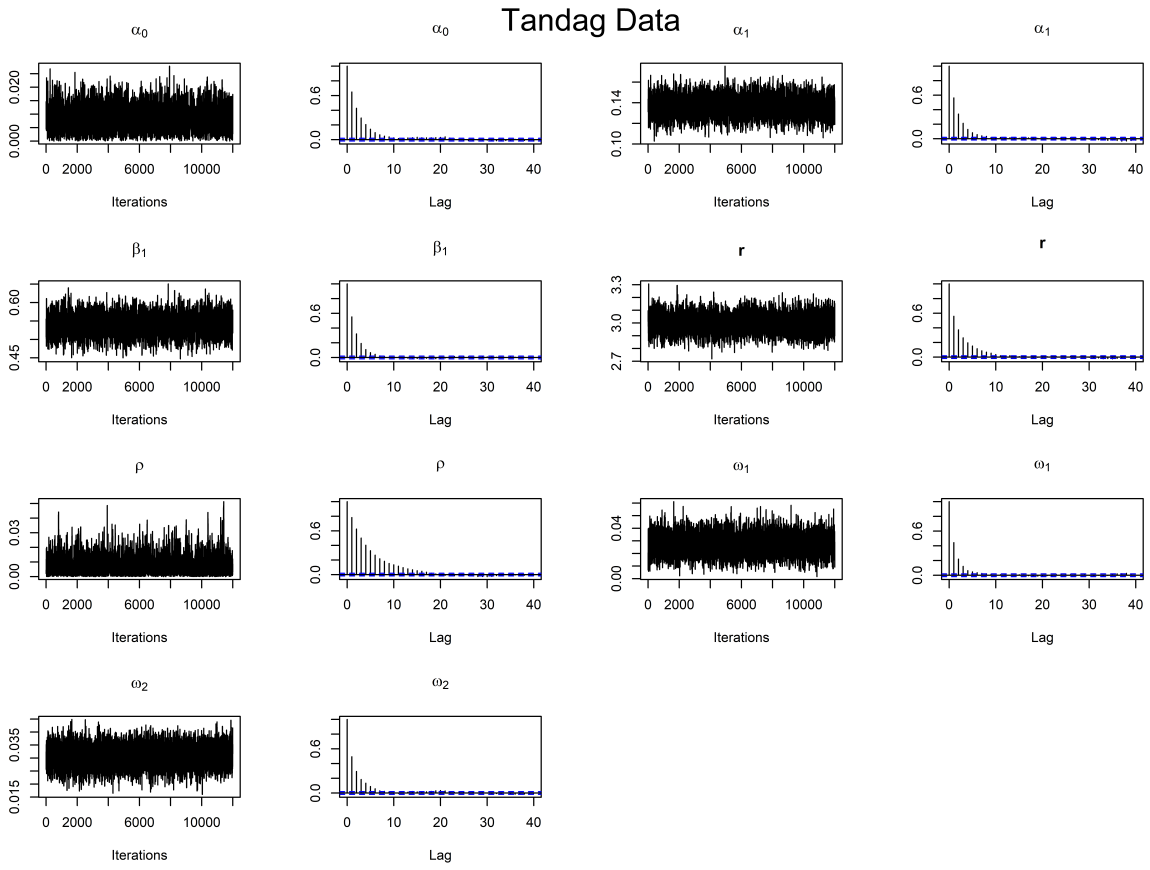

Figure 8: Trace and autocorrelation (ACF) plots for the posterior samples of parameter estimates in the ZINB-INGARCHX model for Tandag City, incorporating two exogenous variables: rainfall and temperature. These plots are used to assess convergence and mixing behavior of the MCMC algorithm.

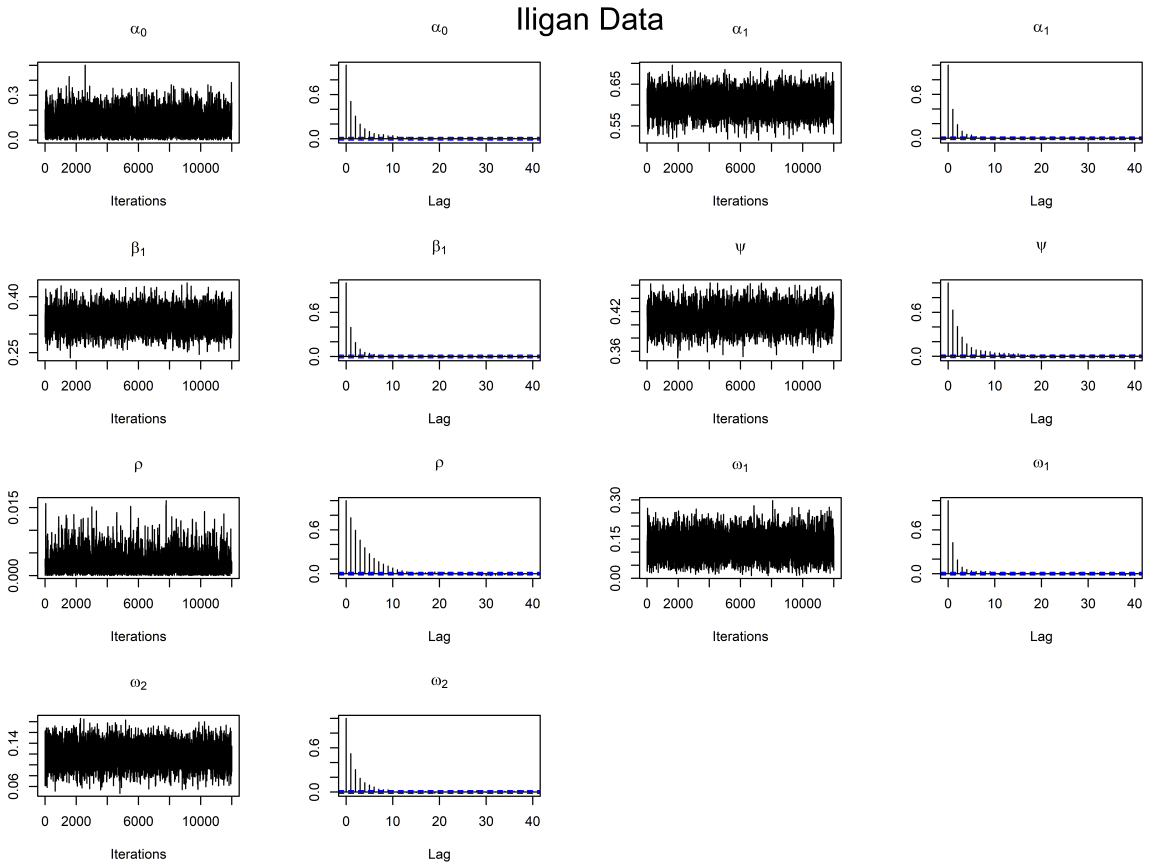

Figure 9: Trace and autocorrelation (ACF) plots for the posterior samples of parameter estimates in the ZIGP-INGARCHX model for Iligan City, incorporating two exogenous variables: rainfall and temperature. These plots are used to assess convergence and mixing behavior of the MCMC algorithm.

Acknowledgements

The authors are grateful to the anonymous referees for their insightful comments and suggestions. They also extend their sincere gratitude to the Department of Science and Technology – Accelerated Science and Technology Human Resource Development Program (DOST-ASTHRDP) for the scholarship support, and to the PRISM – Center for Computational Analytics and Modeling of MSU-IIT for providing essential computational facilities.

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